EFIE-Based Perturbation Analysis of Coupled Microstrip Transmission Lines

by

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ABSTRACT

In this paper, we present an EFIE-based perturbation approximation for solving the eigenmodes of N coupled microstrip lines. The eigenmode current of the isolated line (which is obtained by a Galerkin's MoM solution to the isolated EFIE in an appropriate Chebyshev polynomial series) is used as a zeroth-order perturbation approximation for nearly degenerate eigenmode currents of the loosely-coupled system. In terms of this approximate current, the EFIE's yield an N by N matrix equation which can be solved numerically for the unknown propagation constant ζ. The matrix elements are found to be much more efficient in numerical computation and a large reduction in computation time is achieved.

For validation, the formulation is specialized for the case of two microstrip line coupling. The results of the perturbation approximation are compared with those of the MoM numerical solution, and the validity range of the perturbation approximation is investigated.

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KEY WORDS: Coupled microstrip transmission lines, EFIE, Perturbation approximation, MoM, Eigenmode.

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I. Introduction

The coupling between adjacent, parallel microstrip transmission lines in the micro/mm-wave PC/IC environment was analyzed traditionally with various half-wave and full-wave techniques. In our previous work [1,2], a full-wave electric field integral equation (EFIE) formulation for the currents on N coupled microstrip transmission lines was developed. The geometry of N coupled microstrip lines located in the cover layer at the film/cover interface of a tri-layered conductor/film/cover environment has its EFIE based upon the Sommerfeld-integral representation of an appropriate electric field Green's dyad. Boundary conditions are incorporated in their full generality in the electric Green's function.

To find the eigenmode propagation constants ζ and the associated mode currents, the EFIE was solved by a Galerkin's MoM technique [3,4] with appropriately weighted Chebyshev polynomial basis functions. Though the approach is potentially exact, the direct numerical solution is usually very time consuming. Consequently an approximate but efficient coupled mode perturbation formulation is pursued.

The perturbational method is useful for calculating change in some quantity due to small changes (or perturbations) in the problem [5]. Usually two problems are involved in the procedure of perturbation analysis: the "unperturbed" problem, for which the solution is known, and the "perturbed" problem, which is slightly different from the unperturbed one. In this paper, we present an EFIE-based perturbation approximation to solve for the system eigenmodes of N coupled microstrip lines.

The formulation uses the eigenmode propagation constant of the isolated microstrip line of equal dimensions as a zeroth-order perturbation approximation for nearly-degenerate eigenmode (by nearly-degenerate eigenmode we mean that the propagation constants of the two individual microstrip modes are almost the same) currents of the loosely-coupled (by "loosely coupled", we mean that a small perturbation is introduced

into the coupled microstrips, so that the propagation constant is slightly different from that of the isolated line of equal dimensions) system. In terms of this approximate current, the EFIE's yield an N by N matrix equation which can be solved numerically for the unknown propagation constant ζ. Since the matrix elements are far more efficient in numerical computation, a large reduction in computation time is achieved.

In Section II the mathematical formation is presented. For validation, the formulation is specialized for the analysis of two identical, thin coupled microstrip lines in Section III. Example results in the form of propagation constants and investigation of the validity range of the perturbation approximation are presented in Section IV. We close in Section V with a discussion of further applicability of this technique in the analysis of other coupled integrated waveguiding structures.

II. Formulation

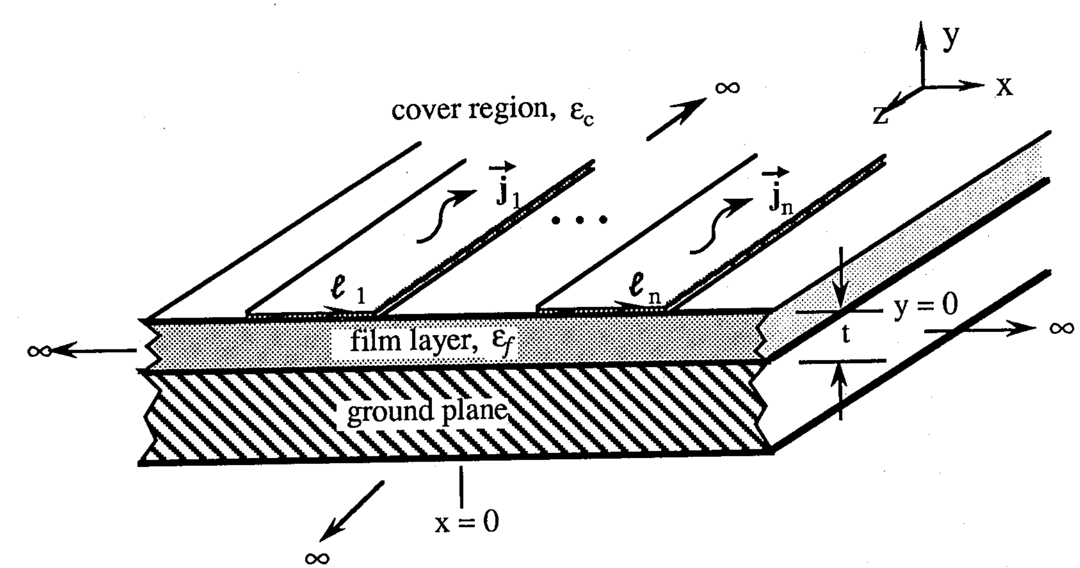

Fig. 1 N coupled microstrip transmission lines.

As a first step of applying this technique to coupled microstrip problems, we write the integral equation for natural modes of N coupled lines shown in Fig. 1 in a more general form as [6]

$$\hat{t}_m \cdot \sum_{n=1}^{N} \int_{\ell_n} \overleftrightarrow{g}^e(\vec{\rho} \mid \vec{\rho}'; \zeta) \cdot \vec{j}_{np}(\vec{\rho}'; \zeta) d\ell' = 0, \quad \vec{\rho} \in \ell_m, \quad m = 1, \ldots, N. \tag{1}$$

where $\vec{j}_{np}(\vec{\rho}'; \zeta)$ represents the current of p^{th} propagation mode along the n^{th} microstrip, and $\overleftrightarrow{g}^e(\vec{\rho} \mid \vec{\rho}'; \zeta)$ is the transformed electric field Green's dyad defined by

$$\overleftrightarrow{g}^e(\vec{\rho} \mid \vec{\rho}'; \zeta) = (k_c^2 + \widetilde{\nabla}\widetilde{\nabla} \cdot) \left[\overleftrightarrow{g}^p(\vec{\rho} \mid \vec{\rho}'; \zeta) + \overleftrightarrow{g}^r(\vec{\rho} \mid \vec{\rho}'; \zeta) \right] \tag{2}$$

with $\overleftrightarrow{g}^p$ and $\overleftrightarrow{g}^r$ given as

$$g^p(\vec{\rho} \mid \vec{\rho}'; \zeta) = \int_{-\infty}^{\infty} \frac{e^{j\xi(x-x')} e^{-p_c|y-y'|}}{4\pi p_c} d\xi \tag{3a}$$

$$\begin{Bmatrix} g_t^r \\ g_n^r \\ g_c^r \end{Bmatrix} = \int_{-\infty}^{\infty} \begin{Bmatrix} R_t \\ R_n \\ C \end{Bmatrix} \frac{e^{j\xi(x-x')} e^{-p_c(y+y')}}{4\pi p_c} d\xi \tag{3b}$$

Following the method of weighted residuals [4], we define the testing operator as

$$\int_{\ell_m} d\ell \, \vec{j}_{mp}^{(0)}(\vec{\rho}; \zeta_{mp}^{(0)}) \cdot \quad , \qquad \vec{\rho} \in \ell_m, \quad m = 1, \ldots, N. \tag{4}$$

where $\vec{j}_{mp}^{(0)}(\vec{\rho}; \zeta_{mp}^{(0)})$ is the current of p^{th} propagation mode along the unperturbed (or isolated) m^{th} microstrip line, and $\zeta_{mp}^{(0)}$ is the p^{th} eigenvalue (or normalized propagation constant) of the unperturbed m^{th} strip associated with $\vec{j}_{mp}^{(0)}$. Then the testing operation of (4) on (1) gives

$$\sum_{n=1}^{N} \int_{\ell_m} d\ell \, \vec{j}_{mp}^{(0)}(\vec{\rho}; \zeta_{mp}^{(0)}) \cdot \int_{\ell_n} \overleftrightarrow{g}^e(\vec{\rho} \mid \vec{\rho}'; \zeta) \cdot \vec{j}_{np}(\vec{\rho}'; \zeta) d\ell' = 0, \quad \vec{\rho} \in \ell_m, \quad m = 1, \ldots, N. \tag{5}$$

where $\hat{t}_m \cdot (\ )$ operator has been embedded in the testing operator.

For the case of loose, nearly-degenerate coupling, the propagation constant and the current distribution are close to those of the unperturbed case, and we can let $\vec{j}_{np}(\vec{\rho}) = a_n \vec{j}_{np}^{(0)}(\vec{\rho})$, where a_n is an unknown constant coefficient. Under this assumption Eq. (5) can be written as

$$\sum_{n=1}^{N} a_n \int_{\ell_m} d\ell \, \vec{j}_{mp}^{(0)}(\vec{\rho}) \cdot \int_{\ell_n} \overleftrightarrow{g}^e(\vec{\rho} \mid \vec{\rho}'; \zeta) \cdot \vec{j}_{np}^{(0)}(\vec{\rho}') \, d\ell' = 0, \quad \vec{\rho} \in \ell_m, \quad m = 1, \ldots, N. \tag{6}$$

It is seen that Eq. (6) has been simplified in evaluation with the known quantity $\vec{j}_{np}^{(0)}(\vec{\rho}')$ retained in the integral and the unknown constant a_n moved to outside. Similarly, we can write $\overleftrightarrow{g}^e(\vec{\rho} \mid \vec{\rho}'; \zeta)$ in terms of its unperturbed counterpart and further simplify the integral on ℓ_n. To do this, we expand $\overleftrightarrow{g}^e(\vec{\rho} \mid \vec{\rho}'; \zeta)$ in Taylor series with respect to $\zeta_{np}^{(0)}$, and retain the first two terms as

$$\overleftrightarrow{g}^e(\vec{\rho} \mid \vec{\rho}'; \zeta) = \overleftrightarrow{g}^e(\vec{\rho} \mid \vec{\rho}'; \zeta_{np}^{(0)}) + \frac{\partial}{\partial \zeta} \overleftrightarrow{g}^e(\vec{\rho} \mid \vec{\rho}'; \zeta) \Big|_{\zeta_{np}^{(0)}} \left(\zeta - \zeta_{np}^{(0)} \right) \tag{7}$$

Then substitution of (7) into Eq. (6) gives

$$\sum_{n=1}^{N} a_n \int_{\ell_m} d\ell \, \vec{j}_{mp}^{(0)}(\vec{\rho}) \cdot \int_{\ell_n} \left[\overleftrightarrow{g}^e(\vec{\rho} \mid \vec{\rho}'; \zeta_{np}^{(0)}) + \frac{\partial \overleftrightarrow{g}^e}{\partial \zeta}(\vec{\rho} \mid \vec{\rho}'; \zeta) \Big|_{\zeta_{np}^{(0)}} (\zeta - \zeta_{np}^{(0)}) \right] \cdot \vec{j}_{np}^{(0)}(\vec{\rho}') d\ell' = 0 \tag{8}$$

where $\vec{\rho} \in \ell_m$ and $m = 1, \ldots, N$.

A close examination of Eq. (8) reveals that we can make use of the isolated EFIE for m^{th} strip, p^{th} mode to simplify the coupled integral equation even further. The current $\vec{j}_{mp}^{(0)}(\vec{\rho}\,')$ on the m^{th} isolated microstrip satisfies the following equation

$$\hat{t}_m \cdot \int_{\ell_m} \overleftrightarrow{g}^e(\vec{\rho}\,|\,\vec{\rho}\,';\, \zeta_{mp}^{(0)}) \cdot \vec{j}_{mp}^{(0)}(\vec{\rho}\,')\, d\ell' = 0 \tag{9}$$

or using the same testing operator as (4), this equation can be written as

$$\int_{\ell_m} d\ell\, \vec{j}_{mp}^{(0)}(\vec{\rho}) \cdot \int_{\ell_m} \overleftrightarrow{g}^e(\vec{\rho}\,|\,\vec{\rho}\,';\, \zeta_{mp}^{(0)}) \cdot \vec{j}_{mp}^{(0)}(\vec{\rho}\,')\, d\ell' = 0 \tag{10}$$

Then by applying (10) to Eq. (8) and retain only the leading non-vanishing terms in the coupled equations, we obtain

$$\overline{C}_{mm}(\zeta - \zeta_{mp}^{(0)})a_m + \sum_{\substack{n=1 \\ m\neq n}}^{N} C_{mn} a_n = 0\,, \qquad m = 1,2,\ldots,N \tag{11}$$

where

$$C_{mn} = \int_{\ell_m} d\ell\, \vec{j}_{mp}^{(0)}(\vec{\rho}) \cdot \int_{\ell_n} \overleftrightarrow{g}^e(\vec{\rho}\,|\,\vec{\rho}\,';\, \zeta_{np}^{(0)}) \cdot \vec{j}_{np}^{(0)}(\vec{\rho}\,')\, d\ell' \tag{12a}$$

and

$$\overline{C}_{mm} = \int_{\ell_m}\int_{\ell_m} \vec{j}_{mp}^{(0)}(\vec{\rho}\,') \cdot \left.\frac{\partial \overleftrightarrow{g}^e}{\partial \zeta}(\vec{\rho}\,|\,\vec{\rho}\,';\, \zeta)\right|_{\zeta_{mp}^{(0)}} \cdot \vec{j}_{mp}^{(0)}(\vec{\rho})\, d\ell'\, d\ell \tag{12b}$$

Equation (11) is an N by N homogeneous matrix equation results from perturbation approximation. Notice that C_{mn} and $\overline{C}_{mm}$ are independent of ζ, therefore the matrix equation is much simpler and more efficient to be solved than that given in Galerkin's MoM.

III. Specialization of the formulation for Two Coupled Microstrips

For the geometry of two coupled microstrip lines depicted in Fig. 2, we solve for the nearly-degenerate case and let ζ_1 and ζ_2 be the propagation constants of the two microstrips under unperturbed condition, respectively. In this case Eq. (11) becomes

$$\begin{pmatrix} \overline{C}_{11}(\zeta - \zeta_1) & C_{12} \\ C_{21} & \overline{C}_{22}(\zeta - \zeta_2) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = 0 \tag{13}$$

This is a matrix equation for the two coupled microstrip line system. It has non-trivial solution for the eigenmode propagation constant ζ which renders the determinant vanish. Solving determinant for zeros, we obtain two shifted coupled-mode propagation constants as

$$\zeta = \overline{\zeta} \pm \sqrt{\Delta^2 + \kappa^2} = \overline{\zeta} \pm \delta \tag{14}$$

where $\overline{\zeta} = (\zeta_1 + \zeta_2)/2$, $\Delta = (\zeta_1 - \zeta_2)/2$, $\kappa^2 = C_{12}C_{21}/\overline{C}_{11}\overline{C}_{22}$ and $\delta = \sqrt{\Delta^2 + \kappa^2}$.

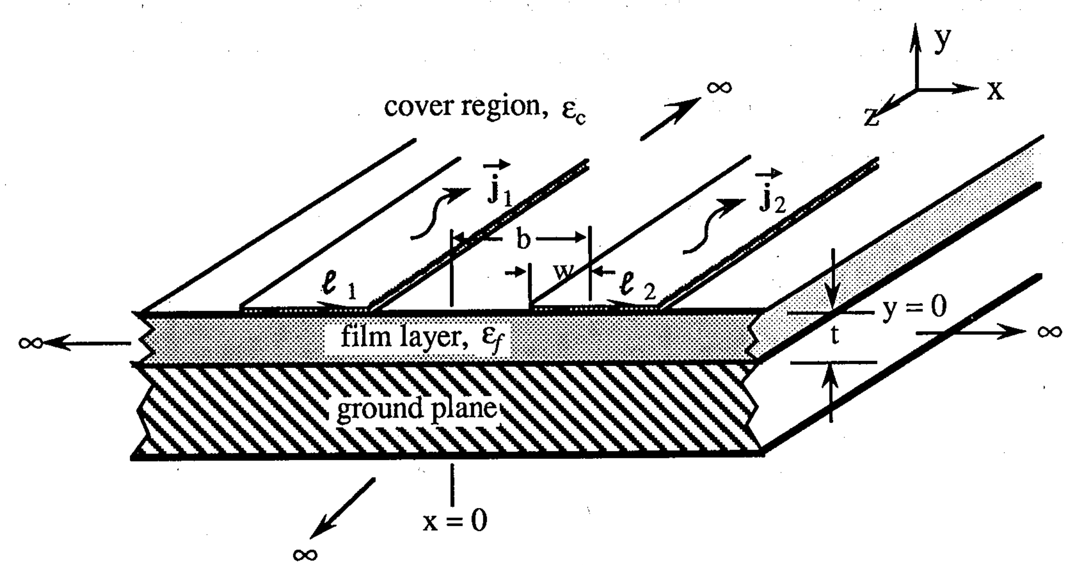

Fig. 2 Two identical thin coupled microstrip lines.

To demonstrate the formulation developed above, we now apply it to solve the special case of loose coupling between fundamental EH_0 modes of the two identical microstrips. Denote the propagation constant of the fundamental mode of the isolated line as ζ_0 and the corresponding current as $\vec{j}_0(\vec{\rho})$. Then for nearly-degenerate case, we have

$$\zeta_1 \approx \zeta_2 \approx \zeta_0$$

and under the assumption of loose coupling, we let

$$\vec{j}_{10}^{(0)}(\vec{\rho}) \approx \vec{j}_{20}^{(0)}(\vec{\rho}) \approx \vec{j}_0(\vec{\rho})$$

It is seen in this case that $C_{12} = C_{21}$, $\overline{C}_{11} = \overline{C}_{22}$ and $\delta = |C_{12}/\overline{C}_{11}|$. So (14) becomes

$$\zeta = \zeta_0 \pm \delta \tag{15}$$

An interpretation for (15) is that the two system modes (the symmetric mode and the antisymmetric mode) can be viewed as emerging and shifting symmetrically from the isolated EH_0 mode of ζ_0. All that remains to be done to find the propagation constant in (15) is the computation of C_{12} and $\overline{C}_{11}$. By Eqs. (12a, b), C_{12} and $\overline{C}_{11}$ have the following forms, respectively

$$C_{12} = \int_{\ell_1} d\ell\, \vec{j}_{10}^{(0)}(\vec{\rho}) \cdot \int_{\ell_2} \overleftrightarrow{g}^{e}(\vec{\rho}\,|\,\vec{\rho}\,';\zeta_{10}^{(0)}) \cdot \vec{j}_{20}^{(0)}(\vec{\rho}\,')\, d\ell' \tag{16a}$$

$$\overline{C}_{11} = \int_{\ell_1}\int_{\ell_1} \vec{j}_{10}^{(0)}(\vec{\rho}\,') \cdot \frac{\partial \overleftrightarrow{g}^{e}}{\partial \zeta}(\vec{\rho}\,|\,\vec{\rho}\,';\zeta)\Big|_{\zeta_{10}^{(0)}} \cdot \vec{j}_{10}^{(0)}(\vec{\rho}\,')\, d\ell'\, d\ell \tag{16b}$$

Before we can perform a direct calculation of C_{12} and $\overline{C}_{11}$, the currents should be expressed in terms of some chosen basis functions as that done in method of moments. The dot product of the currents and the dyadic Green's function should also be evaluated first. The details of this work are included in Appendix. A similar work using only Chebyshev polynomials of the first kind as the expansion bases for both longitudinal and transverse currents has been presented by Y. Yuan et al. [7]. We show here only the final form of C_{12} and $\overline{C}_{11}$ as (see (A.15) and (A.18) in Appendix)

$$C_{12} = \frac{\pi}{4}\left\{ \sum_{m=0}^{N-1} a_m^{(0)} \sum_{n=0}^{N-1} a_n^{(0)} \int_{-\infty}^{\infty} \frac{Y_1}{\xi^2 p_c} e^{-j2\xi b} \left[j^{m-n} (m+1)(n+1) J_{m+1}(w\xi) J_{n+1}(w\xi) \right] d\xi \right.$$
$$\left. + \sum_{m=0}^{N-1} b_m^{(0)} \sum_{n=0}^{N-1} b_n^{(0)} \int_{-\infty}^{\infty} \frac{Y_2}{p_c} e^{-j2\xi b} \left[j^{m-n} w^2 J_m(w\xi) J_n(w\xi) \right] d\xi \right\} \tag{17a}$$

and

$$\overline{C}_{11} = \frac{\pi}{4}\left\{ \sum_{m=0}^{N-1} a_m^{(0)} \sum_{n=0}^{N-1} a_n^{(0)} \int_{-\infty}^{\infty} \frac{(m+1)(n+1)}{\xi^2} j^{m-n} J_{m+1}(w\xi) J_{n+1}(w\xi)\, C_{xx}\, d\xi \right.$$
$$\left. + \sum_{m=0}^{N-1} b_m^{(0)} \sum_{n=0}^{N-1} b_n^{(0)} \int_{-\infty}^{\infty} j^{m-n}\, w^2 J_m(w\xi) J_n(w\xi)\, C_{zz}\, d\xi \right\} \tag{17b}$$

where $Y_1 \equiv k_c^2 R - \xi^2(R - C')$, $Y_2 \equiv k_c^2 R - \zeta_0^2(R - C')$ and C_{xx}, C_{zz} take the forms of (A.20). C_{12} and $\overline{C}_{11}$ in (17) look very complicated in expression, but actually they are much more efficient in numerical computation than that required in the usual MoM procedure.

IV. Example Results

For demonstration of the formulation, an example of coupled lines with dimensions $t/\lambda = 0.021$, $w/t = 2.36$ and $\varepsilon_f = 9.8$ was calculated. A graph showing the normalized propagation constant (ζ/k_0) versus separation between two microstrips is plotted in Fig. 3. The result obtained using EFIE approach in conjunction with MoM (where Galerkin's technique is implemented) is also shown in this figure.

It is seen that as $(b - w)/w \geq 2.5$, the microstrips are separated far enough and the coupling effect is negligible. In this case the coupled mode propagation constant is very close to that of the isolated line of same dimensions. As $(b - w)/w$ becomes smaller, the coupling becomes stronger, and the curves of the two normal modes begin to depart from that of the isolated microstrip.

The graph shows that as long as (b – w)/w is large enough (about 0.5 in this example), the results obtained from perturbation analysis and the more accurate MoM using Galerkin's technique are in good agreement. This makes perfect sense since the accuracy of the perturbation analysis is based on the assumption that only a 'small change' is introduced to the system. For larger perturbation, the approximated method will be less accurate.

The perturbation analysis described in this section can also be applied to solve for higher order modes. The only change to the formulation is the replacement of the propagation constant ζ_0 with those of the higher order modes.

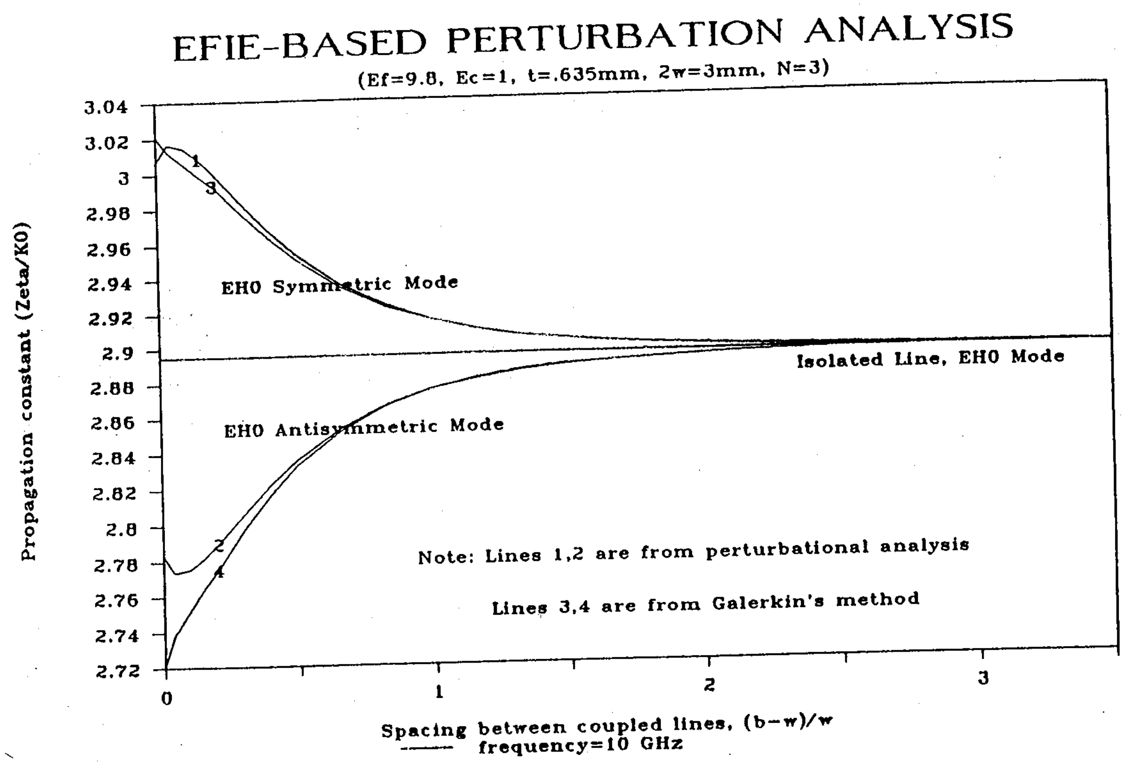

Fig. 3 Dependence of the normalized propagation constant for EH_0 coupled mode on line separation. The parameters used for this coupled structure (see Fig. 2) are $t/\lambda = 0.021$, $w/t = 2.36$ and $\varepsilon_f = 9.8$.

V. Conclusions

In this paper, we presented an EFIE-based perturbation approximation for solving the eigenmodes of N coupled microstrip lines. The example results for the two microstrip line coupling case showed that the propagation constants split and shift symmetrically away from their isolated limit as expected as the two microstrips become closely spaced. This makes perfect sense and the validity of this approach is demonstrated.

The evaluation of the Sommerfeld type integrals is usually very time consuming, which is the major trouble encountered in the analysis of various integrated coupling geometries. The main advantage of utilizing perturbation analysis is that the formulation is far more efficient in numerical computation than those using MoM techniques. This renders a large reduction in computation time. The same approach can be used to analyzed coupled dielectric optical waveguides as far as the propagation constants are all that required.

APPENDIX

Evaluation of C_{12} and $\overline{C}_{11}$ for EH_0 Coupled mode

In this appendix we will perform the detailed procedure for the evaluation of C_{12} and $\overline{C}_{11}$. As the first step, $\overleftrightarrow{g}^e$ in Eqs. (16) is replaced with the expression (2). Then we substitute (3) into Eqs. (16), and assume that the two microstrips are located at y' = 0 so that the following notation of the dyadic Green's function can be applied:

$$g^{p,r}_{t,n,c}(\vec{\rho}\,|\,x';\zeta)=\int_{-\infty}^{\infty} i^{p,r}_{t,n,c}(\vec{\rho}\,|\,x';\zeta,\xi)\,d\xi$$

with

$$i^p = \frac{e^{j\xi(x-x')}e^{-p_c|y|}}{4\pi p_c}$$

$$\begin{Bmatrix} i_t^r \\ i_n^r \\ i_c^r \end{Bmatrix} = \begin{Bmatrix} R_t \\ R_n \\ C \end{Bmatrix} \frac{e^{j\xi(x-x')}e^{-p_c y}}{4\pi p_c}$$

The end result gives

$$C_{12} = \int_{\ell_1} d\ell\, \vec{j}_{10}^{(0)}(\vec{\rho})\cdot \int_{\ell_2}\int_{-\infty}^{\infty} (k_c^2 + \tilde{\nabla}\tilde{\nabla}\cdot)\, \overline{\mathbf{i}}(\vec{\rho}\,|x';\zeta_0,\xi)\cdot \vec{j}_{20}^{(0)}(x')\, d\xi\, d\ell' \qquad \text{(A.1a)}$$

and

$$\overline{C}_{11} = \int_{\ell_1} d\ell \int_{\ell_1} \vec{j}_{10}^{(0)}(\vec{\rho})\cdot \left\{\frac{\partial}{\partial\zeta}\int_{-\infty}^{\infty} (k_c^2 + \tilde{\nabla}\tilde{\nabla}\cdot)\, \overline{\mathbf{i}}(\vec{\rho}\,|x';\zeta,\xi)\, d\xi\right\}\Bigg|_{\zeta_0} \cdot \vec{j}_{10}^{(0)}(x')\, d\ell' \qquad \text{(A.1b)}$$

where $\tilde{\nabla} = j\xi\hat{x} + \frac{\partial}{\partial y}\hat{y} + j\zeta\hat{z}$. For thin microstrips of negligible thickness, we let

$$\vec{j}_{10}^{(0)}(\vec{\rho}) = \vec{j}_{10}^{(0)}(x) = \hat{x}\,\vec{j}_{x1}^{(0)}(x) + \hat{z}\,\vec{j}_{z1}^{(0)}(x) \qquad \text{(A.2a)}$$

and

$$\vec{j}_{20}^{(0)}(\vec{\rho}) = \vec{j}_{20}^{(0)}(x) = \hat{x}\,\vec{j}_{x2}^{(0)}(x) + \hat{z}\,\vec{j}_{z2}^{(0)}(x) \qquad \text{(A.2b)}$$

where the subscript '0' of the current components which represents the fundamental mode EH_0 has been dropped to reduce complexity in notation.

Evaluation of C_{12}

We will first evaluate C_{12}, the procedure for evaluating $\overline{C}_{11}$ is similar. Following the same work work in [6], we obtain for the integrand of the infinite integral in (A.1a) as

$$\tilde{\nabla}\tilde{\nabla}\cdot\ \overline{\mathbf{i}}(\vec{\rho}\,|x';\zeta_0,\xi)\cdot \vec{j}_{20}^{(0)}(x')$$

$$= (j\xi\hat{x} + \frac{\partial}{\partial y}\hat{y} + j\zeta_0\hat{z})(\mathbf{i}^p + \mathbf{i}_t^r - p_c\mathbf{i}_c^r)(j\xi\mathbf{j}_{x2}^{(0)} + j\zeta_0\mathbf{j}_{z2}^{(0)}) \qquad \text{(A.3)}$$

Substitution of $\vec{j}_{10}^{(0)}(\vec{\rho})$ into (A.1a) gives

$$C_{12} = \int_{\ell_1} d\ell \left[\hat{x}\,\vec{j}_{x1}^{(0)}(\vec{\rho}) + \hat{z}\,\vec{j}_{z1}^{(0)}(\vec{\rho})\right]\cdot \int_{\ell_2}\int_{-\infty}^{\infty} (k_c^2 + \tilde{\nabla}\tilde{\nabla}\cdot)\, \overline{\mathbf{i}}\cdot \vec{j}_{20}^{(0)}(x')\, d\xi\, dx' \qquad \text{(A.4)}$$

Then applying (A.3) to above equation, the $\hat{x}$ and $\hat{y}$ dot product terms can be written as

$$\hat{x}\,\mathbf{j}_{x1}^{(0)}(\vec{\rho})\cdot (k_c^2 + \tilde{\nabla}\tilde{\nabla}\cdot)\, \overline{\mathbf{i}}(\vec{\rho}\,|x';\zeta_0,\xi)\cdot \vec{j}_{20}^{(0)}(x')$$

$$= \mathbf{j}_{x1}^{(0)}(\vec{\rho})\left[k_c^2(\mathbf{i}^p + \mathbf{i}_t^r)\mathbf{j}_{x2}^{(0)}(x') + j\xi(\mathbf{i}^p + \mathbf{i}_t^r - p_c\mathbf{i}_c^r)(j\xi\mathbf{j}_{x2}^{(0)} + j\zeta_0\mathbf{j}_{z2}^{(0)})\right] \qquad \text{(A.5)}$$

and

$$\hat{z}\,\mathbf{j}_{z1}^{(0)}(\vec{\rho})\cdot (k_c^2 + \tilde{\nabla}\tilde{\nabla}\cdot)\, \overline{\mathbf{i}}(\vec{\rho}\,|x';\zeta_0,\xi)\cdot \vec{j}_{20}^{(0)}(x')$$

$$= \mathbf{j}_{z1}^{(0)}(\vec{\rho})\left[k_c^2(\mathbf{i}^p + \mathbf{i}_t^r)\mathbf{j}_{z2}^{(0)}(x') + j\zeta_0(\mathbf{i}^p + \mathbf{i}_t^r - p_c\mathbf{i}_c^r)(j\xi\mathbf{j}_{x2}^{(0)} + j\zeta_0\mathbf{j}_{z2}^{(0)})\right] \qquad \text{(A.6)}$$

Based on the same discussion addressed in [6] we see that (A.5) and (A.6) also pose no problem while taking the limit $y \to 0$. Thus the right hand side of (A.5), after taking $y \to 0$ and collecting terms, becomes

$$\mathbf{j}_{x1}^{(0)}(x)\,\frac{e^{j\xi(x-x')}}{4\pi p_c}\left\{[k_c^2 R - \xi^2(R - C')]\,\mathbf{j}_{x2}^{(0)}(x') - \xi\zeta_0(R - C')\mathbf{j}_{z2}^{(0)}(x')\right\} \qquad \text{(A.7)}$$

where

$$\lim_{y\to 0}\,(\mathbf{i}^p + \mathbf{i}_t^r) = \frac{e^{j\xi(x-x')}}{4\pi p_c}R$$

with $R = 1 + R_t$, and

$$\lim_{y\to 0}\,(p_c\,\mathbf{i}_c^r) = \frac{e^{j\xi(x-x')}}{4\pi p_c}C'$$

with $C' = p_c C$ have been applied. Similarly, the right hand side of (A.6) becomes

$$\mathbf{j}_{z1}^{(0)}(x)\,\frac{e^{j\xi(x-x')}}{4\pi p_c}\left\{[k_c^2 R - \zeta_0^2(R - C')]\,\mathbf{j}_{z2}^{(0)}(x') - \xi\zeta_0(R - C')\mathbf{j}_{x2}^{(0)}(x')\right\} \qquad \text{(A.8)}$$

Substituting (A.7) and (A.8) back to (A.4), we obtain

$$C_{12} = \int_{-b-w}^{-b+w} dx\, \mathbf{j}_{x1}^{(0)}(x) \int_{b-w}^{b+w} \int_{-\infty}^{\infty} \frac{e^{j\xi(x-x')}}{4\pi p_c} \left[Y_1\, \mathbf{j}_{x2}^{(0)}(x') - Y_3\, \mathbf{j}_{z2}^{(0)}(x') \right] d\xi dx'$$

$$+ \int_{-b-w}^{-b+w} dx\, \mathbf{j}_{z1}^{(0)}(x) \int_{b-w}^{b+w} \int_{-\infty}^{\infty} \frac{e^{j\xi(x-x')}}{4\pi p_c} \left[Y_2\, \mathbf{j}_{z2}^{(0)}(x') - Y_3\, \mathbf{j}_{x2}^{(0)}(x') \right] d\xi dx' \qquad \text{(A.9)}$$

where $Y_1 \equiv k_c^2 R - \xi^2(R - C')$, $Y_2 \equiv k_c^2 R - \zeta_0^2(R - C')$ and $Y_3 \equiv \xi\zeta_0(R - C')$. The integration limits have been specified more specifically by replacing ℓ_1 with $[-b-w, -b+w]$ and ℓ_2 with $[b-w, b+w]$ in the above expression.

For currents on the isolated microstrip and on each of the coupled microstrips, we can approximate them as follows

$$\mathbf{j}_x(x) \cong \sqrt{1 - \left(\frac{\alpha}{w}\right)^2} \sum_{n=0}^{N-1} a_n^{(i)} U_n\left(\frac{\alpha}{w}\right) \qquad \text{(A.10a)}$$

$$-w \le \alpha \le w$$

$$\mathbf{j}_z(x) \cong \frac{1}{\sqrt{1 - \left(\frac{\alpha}{w}\right)^2}} \sum_{n=0}^{N-1} b_n^{(i)} T_n\left(\frac{\alpha}{w}\right) \qquad \text{(A.10b)}$$

where $\alpha = x$, $i = 0$ for isolated microstrip, $\alpha = x + b$, $i = 1$ for microstrip 1 in Fig. 2, and $\alpha = x - b$, $i = 2$ for microstrip 2 in the same coupled line system. So if we change variable to let $u = x + b$ and $v = x - b$, then all the three expansion forms will be the same (i.e. $\mathbf{j}_1(u) = \mathbf{j}_2(v) = \mathbf{j}_0^{(0)}(x)$). For example, for the unperturbed EH_0 microstrip mode, we have $\mathbf{j}_{10}^{(0)}(u) = \mathbf{j}_{20}^{(0)}(v) = \mathbf{j}^{(0)}(x)$.

Based on the above expression of currents, let us examine the two cross-terms of C_{12} in (A.9) one step further. They are rewritten here for convenience as

$$-\int_{-b-w}^{-b+w} dx\, \mathbf{j}_{x1}^{(0)}(x) \cdot \int_{b-w}^{b+w} \int_{-\infty}^{\infty} \frac{e^{j\xi(x-x')}}{4\pi p_c} Y_3\, \mathbf{j}_{z2}^{(0)}(x')\, d\xi dx'$$

and

$$-\int_{-b-w}^{-b+w} dx\, \mathbf{j}_{z1}^{(0)}(x) \cdot \int_{b-w}^{b+w} \int_{-\infty}^{\infty} \frac{e^{j\xi(x-x')}}{4\pi p_c} Y_3\, \mathbf{j}_{x2}^{(0)}(x')\, d\xi dx'$$

Then by changing variables and carrying out the dot operations, we obtain

$$-\int_{-\infty}^{\infty} \frac{Y_3}{4\pi p_c} e^{-j2\xi b} \int_{-w}^{+w} e^{j\xi u}\, \mathbf{j}_x^{(0)}(u)\, du \int_{-w}^{+w} e^{-j\xi v}\, \mathbf{j}_z^{(0)}(v)\, dv\, d\xi$$

and

$$-\int_{-\infty}^{\infty} \frac{Y_3}{4\pi p_c} e^{-j2\xi b} \int_{-w}^{+w} e^{j\xi u}\, \mathbf{j}_z^{(0)}(u)\, du \int_{-w}^{+w} e^{-j\xi v}\, \mathbf{j}_x^{(0)}(v)\, dv\, d\xi$$

For antisymmetric modes, $\mathbf{j}_x^{(0)}$ is even and $\mathbf{j}_z^{(0)}$ is odd. By making use of the symmetry characteristics, the above terms can be reduced to

$$-\int_{-\infty}^{\infty} \mathrm{Int}(\xi) \int_{-w}^{+w} (\cos \xi u)\, \mathbf{j}_x^{(0)}(u)\, du \int_{-w}^{+w} (-j \sin \xi v)\, \mathbf{j}_z^{(0)}(v)\, dv\, d\xi$$

and

$$-\int_{-\infty}^{\infty} \mathrm{Int}(\xi) \int_{-w}^{+w} (j \sin \xi u)\, \mathbf{j}_z^{(0)}(u)\, du \int_{-w}^{+w} (\cos \xi u)\, \mathbf{j}_x^{(0)}(v)\, dv\, d\xi$$

So they are in the same form but of different signs, and will cancel out in the calculation. Notice that we have denoted $\mathrm{Int}(\xi)$ as

$$\mathrm{Int}(\xi) = \frac{Y_3}{4\pi p_c} e^{-j2\xi b}$$

Similarly, for symmetric modes, $\mathbf{j}_x^{(0)}$ is odd and $\mathbf{j}_z^{(0)}$ is even. Thus the two cross terms can be reduced to

$$-\int_{-\infty}^{\infty} \mathrm{Int}(\xi) \int_{-w}^{+w} (j \sin \xi u)\, \mathbf{j}_x^{(0)}(u)\, du \int_{-w}^{+w} (\cos \xi v)\, \mathbf{j}_z^{(0)}(v)\, dv\, d\xi$$

and

$$-\int_{-\infty}^{\infty} \mathrm{Int}(\xi) \int_{-w}^{+w} (\cos \xi u)\, j_z^{(0)}(u)\, du \int_{-w}^{+w} (-j \sin \xi v)\, j_x^{(0)}(v)\, dv\, d\xi$$

Again they have the same form but of different signs and will add up to zero.

With the two cross-terms removed, C_{12} now becomes

$$C_{12} = \int_{-\infty}^{\infty} \frac{Y_1}{4\pi p_c} \int_{-b-w}^{-b+w} j_{x1}^{(0)}(x)\, e^{j\xi x}\, dx \int_{b-w}^{b+w} j_{x2}^{(0)}(x) e^{j\xi x'}\, dx' d\xi$$
$$+ \int_{-\infty}^{\infty} \frac{Y_2}{4\pi p_c} \int_{-b-w}^{-b+w} j_{z1}^{(0)}(x)\, e^{j\xi x}\, dx \int_{b-w}^{b+w} j_{z2}^{(0)}(x)\, e^{j\xi x'}\, dx' d\xi \qquad \text{(A.11)}$$

Substitution of expression (A.10) into (A.11) with appropriate arguments for $j_{10}^{(0)}$ and $j_{20}^{(0)}$, we obtain

$$C_{12} = \sum_{m=0}^{N-1} a_m^{(0)} \sum_{n=0}^{N-1} a_n^{(0)} \int_{-\infty}^{\infty} \frac{Y_1}{4\pi p_c} f_{xm1}^{(+)}(x)\, f_{xn2}^{(-)}(x') d\xi$$
$$+ \sum_{m=0}^{N-1} b_m^{(0)} \sum_{n=0}^{N-1} b_n^{(0)} \int_{-\infty}^{\infty} \frac{Y_2}{4\pi p_c} f_{zm1}^{(+)}(x)\, f_{zn2}^{(-)}(x') d\xi \qquad \text{(A.12)}$$

where

$$f_{xn1}^{(\pm)}(x) \equiv \int_{-b-w}^{-b+w} e^{\pm j\xi x} \sqrt{1 - \left(\frac{x+b}{w}\right)^2}\; U_n\left(\frac{x+b}{w}\right) dx \qquad \text{(A.13a)}$$

$$f_{xn2}^{(\pm)}(x) \equiv \int_{b-w}^{b+w} e^{\pm j\xi x} \sqrt{1 - \left(\frac{x-b}{w}\right)^2}\; U_n\left(\frac{x-b}{w}\right) dx \qquad \text{(A.13b)}$$

$$f_{zn1}^{(\pm)}(x) \equiv \int_{-b-w}^{-b+w} e^{\pm j\xi x}\, T_n\left(\frac{x+b}{w}\right) \frac{dx}{\sqrt{1 - \left(\frac{x-b}{w}\right)^2}} \qquad \text{(A.13c)}$$

$$f_{zn2}^{(\pm)}(x) \equiv \int_{b-w}^{b+w} e^{\pm j\xi x}\, T_n\left(\frac{x-b}{w}\right) \frac{dx}{\sqrt{1 - \left(\frac{x-b}{w}\right)^2}} \qquad \text{(A.13d)}$$

To evaluate (C.13), we make variable changes by letting u' = (x + b)/w and v' = (x' − b)/w, and then refer to the formulas developed in Appendix B of [4]. We obtain

$$f_{xn1}^{(\pm)}(x) = e^{\mp j\xi b} \begin{Bmatrix} j^n \\ (j^n)^* \end{Bmatrix} \frac{\pi(n+1)}{\xi} J_{n+1}(\xi w) \qquad \text{(A.14a)}$$

$$f_{xn2}^{(\pm)}(x) = e^{\pm j\xi b} \begin{Bmatrix} j^n \\ (j^n)^* \end{Bmatrix} \frac{\pi(n+1)}{\xi} J_{n+1}(\xi w) \qquad \text{(A.14b)}$$

$$f_{zn1}^{(\pm)}(x) = e^{\mp j\xi b} \begin{Bmatrix} j^n \\ (j^n)^* \end{Bmatrix} w\pi J_n(\xi w) \qquad \text{(A.14c)}$$

$$f_{zn2}^{(\pm)}(x) = e^{\pm j\xi b} \begin{Bmatrix} j^n \\ (j^n)^* \end{Bmatrix} w\pi J_n(\xi w) \qquad \text{(A.14d)}$$

where $(j^n)^*$ represents the complex conjugate of j^n and J_n , J_{n+1} are Bessel functions of order n and n + 1, respectively. Applying (A.14) to (A.12), we obtain the final form of C_{12} as

$$C_{12} = \frac{\pi}{4} \left\{ \sum_{m=0}^{N-1} a_m^{(0)} \sum_{n=0}^{N-1} a_n^{(0)} \int_{-\infty}^{\infty} \frac{Y_1}{\xi^2 p_c} e^{-j2\xi b} \left[j^{m-n} (m+1)(n+1)\, J_{m+1}(w\xi)\, J_{n+1}(w\xi) \right] d\xi \right.$$
$$\left. + \sum_{m=0}^{N-1} b_m^{(0)} \sum_{n=0}^{N-1} b_n^{(0)} \int_{-\infty}^{\infty} \frac{Y_2}{p_c} e^{-j2\xi b} \left[j^{m-n} w^2 J_m(w\xi)\, J_n(w\xi) \right] d\xi \right\} \qquad \text{(A.15)}$$

<u>Evaluation of $\overline{C}_{11}$</u>

Refer to (A.1b). Since $\vec{j}_{10}^{(0)}$ is known for the fundamental coupled microstrip mode EH_0, it is independent of ζ, and can be moved inside the differentiation operator $\partial/\partial\zeta$. So (A.1b) can be rewritten as

$$\overline{C}_{11} = \int_{\ell_1} d\ell \int_{\ell_1} \left\{ \frac{\partial}{\partial\zeta} \int_{-\infty}^{\infty} \vec{j}_{10}^{(0)}(\vec{\rho}) \cdot \left(k_c^2 + \tilde{\tilde{\nabla}}\tilde{\tilde{\nabla}} \cdot \right) \bar{\mathbf{i}}(\vec{\rho}\,|x';\zeta,\xi) \cdot \vec{j}_{10}^{(0)}(x')\,d\xi \right\} \Big|_{\zeta_0} d\ell'$$

Then by using (A.2a) and (A.3) to above expression of $\overline{C}_{11}$ and carrying out the dot operation, we obtain

$$\overline{C}_{11} =$$

$$\int_{\ell_1} d\ell \int_{\ell_1} \int_{-\infty}^{\infty} \frac{\partial}{\partial\zeta} \left\{ \mathbf{j}_{x1}^{(0)}(\vec{\rho}) \left[k_c^2(\mathbf{i}^p + \mathbf{i}_t^r)\mathbf{j}_{x1}^{(0)}(x') + j\xi(\mathbf{i}^p + \mathbf{i}_t^r - p_c\mathbf{i}_c^r)(j\xi\mathbf{j}_{x1}^{(0)} + j\zeta_0\mathbf{j}_{z1}^{(0)}) \right] \right\} \Big|_{\zeta_0} d\xi\, dx'$$

$$+ \int_{\ell_1} d\ell \int_{\ell_1} \int_{-\infty}^{\infty} \frac{\partial}{\partial\zeta} \left\{ \mathbf{j}_{z1}^{(0)}(\vec{\rho}) \left[k_c^2(\mathbf{i}^p + \mathbf{i}_t^r)\mathbf{j}_{z1}^{(0)}(x') + j\zeta_0(\mathbf{i}^p + \mathbf{i}_t^r - p_c\mathbf{i}_c^r)(j\xi\mathbf{j}_{x1}^{(0)} + j\zeta_0\mathbf{j}_{z1}^{(0)}) \right] \right\} \Big|_{\zeta_0} d\xi\, dx' \tag{A.16}$$

As in the evaluation of C_{12}, we take the limit $y \to 0$ of $\overline{C}_{11}$, then

$$\overline{C}_{11} = \int_{-b-w}^{-b+w} dx\, \mathbf{j}_{x1}^{(0)}(x) \int_{-b-w}^{-b+w} \int_{-\infty}^{\infty} \frac{\partial}{\partial\zeta} \left\{ \frac{e^{j\xi(x-x')}}{4\pi p_c} \left[Y_1\,\mathbf{j}_{x1}^{(0)}(x') - Y_3(\zeta)\,\mathbf{j}_{z1}^{(0)}(x') \right] \right\} \Big|_{\zeta_0} d\xi\, dx'$$

$$+ \int_{-b-w}^{-b+w} dx\, \mathbf{j}_{z1}^{(0)}(x) \int_{-b-w}^{-b+w} \int_{-\infty}^{\infty} \frac{\partial}{\partial\zeta} \left\{ \frac{e^{j\xi(x-x')}}{4\pi p_c} \left[Y_2(\zeta)\,\mathbf{j}_{z1}^{(0)}(x') - Y_3(\zeta)\,\mathbf{j}_{x1}^{(0)}(x') \right] \right\} \Big|_{\zeta_0} d\xi\, dx' \tag{A.17}$$

where $Y_2(\zeta)$ and $Y_3(\zeta)$ represent Y_2 and Y_3 with ζ_0 replaced by ζ.

By the same procedure as that for C_{12}, we can show that the cross-terms in (A.17) also add up to zero and (A.17) can be further simplified. Now we expand $\vec{j}_{10}^{(0)}$ in the form of (A.10) and applying expressions (A.13) to (A.17) after expansion, $\overline{C}_{11}$ results in the form as

$$\overline{C}_{11} = \sum_{m=0}^{N-1} a_m^{(0)} \sum_{n=0}^{N-1} a_n^{(0)} \int_{-\infty}^{\infty} \frac{f_{xm1}^{(+)}(x)\, f_{xn1}^{(-)}(x')}{4\pi} \frac{\partial}{\partial\zeta} \left\{ \frac{Y_1}{p_c} \right\} \Big|_{\zeta_0} d\xi$$

$$+ \sum_{m=0}^{N-1} b_m^{(0)} \sum_{n=0}^{N-1} b_n^{(0)} \int_{-\infty}^{\infty} \frac{f_{zm1}^{(+)}(x)\, f_{zn1}^{(-)}(x')}{4\pi} \frac{\partial}{\partial\zeta} \left\{ \frac{Y_2(\zeta)}{p_c} \right\} \Big|_{\zeta_0} d\xi \tag{A.18}$$

or by formulas (A.14),

$$\overline{C}_{11} = \frac{\pi}{4} \left\{ \sum_{m=0}^{N-1} a_m^{(0)} \sum_{n=0}^{N-1} a_n^{(0)} \int_{-\infty}^{\infty} \frac{(m+1)(n+1)}{\xi^2} j^{m-n} J_{m+1}(w\xi) J_{n+1}(w\xi)\, C_{xx}\, d\xi \right.$$

$$\left. + \sum_{m=0}^{N-1} b_m^{(0)} \sum_{n=0}^{N-1} b_n^{(0)} \int_{-\infty}^{\infty} j^{m-n}\, w^2 J_m(w\xi) J_n(w\xi)\, C_{zz}\, d\xi \right\} \tag{A.19}$$

where

$$C_{xx} \equiv \frac{\partial}{\partial\zeta} \left\{ \frac{Y_1}{p_c} \right\} \Big|_{\zeta_0} \tag{A.20a}$$

and

$$C_{zz} \equiv \frac{\partial}{\partial\zeta} \left\{ \frac{Y_2(\zeta)}{p_c} \right\} \Big|_{\zeta_0} \tag{A.20b}$$

The expressions (A.15) and (A.18) are the final forms for C_{12} and $\overline{C}_{11}$, respectively, which can be computed using general numerical quadrature methods.

REFERENCES

[1] J. S. Bagby, C.-H. Lee, Y. Yuan, and D. P. Nyquist, "Entire-domain basis MoM analysis of coupled microstrip transmission lines," IEEE Trans. Microwave Theory Tech., vol. 40, no. 1, Jan. 1992, pp.49-57.

[2] C. -H. Lee and J. S. Bagby, "The Analysis of Coupled Microstrip Transmission Lines with EFIE Method," 1988 URSI Radio Science Meeting, Program and Abstracts, Jun. 1988, p.318.

[3] Constantine A. Balanis, Antenna Theory, Harper & Row, New York, 1982, p.308.

[4] Warren L. Stutzman and Gary A. Thiele, Antenna Theory and Design, John Wiley & Sons, New York, 1981, pp. 310-315.

[5] R. F. Harrington, Time-Harmonic Electromagnetic Fields, New York, NY: McGraw-Hill, Inc., 1961, pp.317-338.

[6] 涂佳君、李清和, "Integral Equation Analysis of Coupled Microstrip Transmission Lines," The 7th T.V.E. Conference of R.O.C., Mar. 1992, pp.223-232.

[7] Yi Yuan and D. P. Nyquist, "EFIE-Based perturbation approximation for coupled lines," Nat. Radio Sci. Meeting, University of Colorado, Boulder, Colorado, Jan. 1989, digest p.180.

以EFIE爲基礎之耦合微帶傳輸線擾動近似法解析

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摘　　要

本文旨在介紹一以電場型積分方程爲基礎之擾動近似法來解析N條耦合微帶傳輸線之傳播模態。在公式推導過程中，(由Galerkin之MoM法所解得的)單條微帶之模態電流被用作疏耦合微帶系統電流之零階近似。由此，EFIE可以寫成以此近似電流爲變數之矩陣方程，並可藉由簡單之數值計算以求得特徵模態之傳播常數ζ。由於各矩陣元素之數值計算相當快速，故可大大地降低電腦計算之時間。

爲了驗証本方法之實用性，本研究所導出之N條耦合微帶系統之擾動近似分析法公式將被簡化，以用來解析兩條微帶之耦合結構，其結果並將與Galerkin之MoM法所解得者做一比較。同時，亦將對此擾動近似法之適用範圍做一討論。

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