Integral Equation Formulation of Microstrip Resonators of Arbitrary Shape

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Abstract

A spectral-domain, full-wave technique based on an electric field integral equation formulation is developed for the analysis of laterally open microstrip-patch resonators of arbitrary shape. The effects of the layered background structure are rigorously incorporated in the formulation by means of the Hertzian vector potential Green's function. For numerical solution, a Galerkin's method of moments is implemented. The conducting patch is modeled using planar triangular surface patches, and the current distribution on the microstrip patch is approximated in terms of subdomain-type vector basis functions defined over the triangular elements.

For validation of this approach, the modeling of an example rectangular shape microstrip resonator is presented in detail, and the computed results will be published in a coming paper.

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I. Introduction

Printed microstrip-patch resonators of various shapes are used widely at microwave and millimeter-wave frequencies for building filters, oscillators, antennas, etc. These resonators have narrow bandwidths and can only operate effectively in the vicinity of the resonant frequency. Therefor, an efficient and accurate approach to help ascertain the resonant frequency is important in design work. A number of theoretical methods have been advanced for the analysis of microstrip resonant structures, the most well-known and accurate techniques are the rigorous full-wave spectral or space domain methods which account for all wave phenomena associated with the structures and the layered background medium [1]–[7]. Though the full-wave approaches are usually very complicated, it was seen in the literature that the results by the full-wave analysis agree extremely well with those measured at high frequencies [8].

In this paper, we present a full-wave technique based upon a rigorous spectral domain formulation of electric field integral equation (EFIE) for the analysis of microstrip-patch resonators. Because the EFIE formulation is utilized, the procedure is applicable to both open and closed resonator surfaces.

For numerical solution, the patch or resonator surface is modeled using certain kind of surfaces patches. In this research, a set of special subdomain-type vector triangular basis functions which are defined on elemental triangular patches is used in the Galerkin's method of moments procedure to represent the unknown patch current. A crucial characteristic of these triangular functions is that they yield a current representation free of line or point charges at subdomain boundaries. Another important point of these vector basis functions is that they are easily extendible to approximate the currents of microstrip patch resonators of arbitrary shape.

In Section II, a brief review of the EFIE formulation for a general microstrip resonator is presented. The vector triangular basis functions for modeling the current on the microstrip patch is developed in Section III. To solve the EFIE, the Galerkin's moment method is adopted in conjunction with the vector basis functions defined over triangular subdomains to obtain a linear system of equations for the patch current. In Section IV, we give an example of vector triangular basis modeling for a rectangular perfect electric conducting patch. The validation of this approach and areas of applications are presented in Section V.

II. The EFIE Formulation for a General Micorstrip Resonator

Consider the integrated microstrip patch of arbitrary shape depicted in Fig.1. The integral equation satisfied by the unknown surface current $\vec{J}_s$ on the patch is (for natural modes)[9]

$$\hat{t}\cdot\left(k_c^2+\nabla\nabla\cdot\right)\int_s \overleftrightarrow{G}(\vec{r}\,|\,\vec{r}\,')\cdot\vec{J}_s(\vec{\rho}\,')\,ds'=0\ ,\quad \vec{\rho}\,'\in s \tag{1}$$

where $\overleftrightarrow{G}(\vec{r}\,|\,\vec{r}\,')$ is the electric Hertzian potential Green's function for the planar strip in the layered dielectric-conductor background structure shown in Fig.1. The Green's

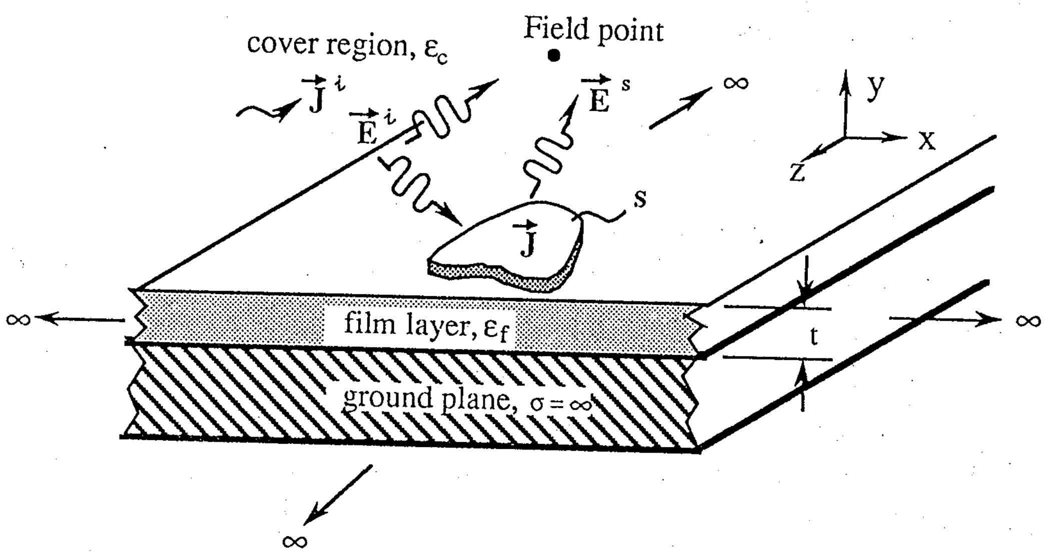

Fig. 1 A general microstrip resonator.

function is in dyadic form and decomposes into a principal and a reflected part, $\overleftrightarrow{G}(\vec{\rho}\,|\,\vec{\rho}\,')=\overleftrightarrow{G}^{p}+\overleftrightarrow{G}^{r}$, where $\overleftrightarrow{G}^{p}$ is the two-dimensional unbounded space Green's function , and the reflected Green's function has components

$$\overleftrightarrow{G}^{r}=\hat{x}G_t^r\hat{x}+\hat{y}\left(\frac{\partial}{\partial x}G_c^r\,\hat{x}+G_n^r\,\hat{y}+\frac{\partial}{\partial z}G_c^r\,\hat{z}\right)+\hat{z}G_t^r\,\hat{z} \tag{2}$$

The scalar components of the reflected Green's dyad are given in terms of inverse transform integrals as

$$\begin{Bmatrix} G_t^r \\ G_n^r \\ G_c^r \end{Bmatrix}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\begin{Bmatrix} R_t \\ R_n \\ C \end{Bmatrix}\frac{e^{j\xi(x-x')}e^{j\zeta(z-z')}e^{-p_c(y+y')}}{4\pi\,p_c}\,d\xi\,d\zeta \tag{3}$$

where $\lambda^2=\xi^2+\zeta^2$, and the wavenumber parameters are defined as $p_c^2=\lambda^2-k_c^2$, and $p_f^2=\lambda^2-k_f^2$, where k_c and k_f are the wavenumbers for the cover region and dielectric film layer, respectively. The reflection and coupling coefficients in the integrands of the reflected Green's dyad components are given as

$$R_t=\frac{p_c-p_f\coth(p_f t)}{p_c+p_f\coth(p_f t)}$$

$$R_n=\frac{Kp_c-p_f\tanh(p_f t)}{Kp_c+p_f\tanh(p_f t)} \tag{4}$$

$$C=\frac{2(K-1)p_c}{[p_c+p_f\coth(p_f t)]\,[Kp_c+p_f\tanh(p_f t)]}$$

where $K=\varepsilon_f/\varepsilon_c$, and ε_c and ε_f are the permittivities of the cover region and the film layer respectively.

The integrands in the spectral representation of Green's dyad components contain multi-valued parameters p_c and p_f, necessitating a choice of branch cuts in the complex ξ plane and ζ plane. The integrands of the reflected Green's dyad also contain the coefficients R_t, R_n, and C. The first one has a simple pole when $\lambda=\lambda_p$ is an eigenvalue of a TE mode of the symmetric slab background structure, while the second coefficient has a simple pole when $\lambda=\lambda_p$ is an eigenvalue of a TM mode of the symmetric slab background structure, and the third one has simple poles at both TE and TM eigenvalues. Careful examination of the integrands reveals that all are even functions of p_f, so the branch cut for this parameter is not implicated.

III. MOM technique

A. Equivalent current representation with planar triangular patch expansions

To solve Eq.(1) numerically, we model the microstrip patch by triangular elements. The nodes of each triangular element are assigned the indices i, j and k in the counterclockwise direction, as illustrated in Fig.2. Here we adopt a local indexing scheme, in which these indices assume the values 1, 2, or 3 in a cyclic manner. The sides of any elemental triangle T^n are formed by three edge vectors $\vec{\ell}_i^{(n)}$, $i = 1, 2, 3$, with $\vec{\ell}_i^{(n)}$ oriented from node j to node k. Partitioning T^n into three parts $A_i^{(n)}$, the total area $A^{(n)}$ is thus $\sum_{i=1}^{3} A_i^{(n)}$. We then define L_i as the area coordinate associated with the ith node [10], which is given as

$$L_i = \frac{A_i^{(n)}}{A^{(n)}} \quad , \quad \sum_{i=1}^{3} L_i = 1 \tag{5}$$

The global position vector $\vec{r}$ of an arbitrary point x on T^n can be expressed in terms of the area coordinate and the local position vector $\vec{\rho}_i^{(n)}$ as

$$\vec{r} = \vec{r}_i^{(n)} + \vec{\rho}_i^{(n)} \quad , \quad \vec{\rho}_i^{(n)} = \vec{\ell}_k^{(n)} L_j + \vec{\ell}_j^{(n)} L_k \quad , \quad i = 1, 2, 3 \tag{6}$$

On modeling the microstrip patch, the patch current on T^n is approximated as

$$\vec{J}_s^{(n)}(\vec{\rho}) = \sum_{i=1}^{3} I_i^{(n)} \vec{V}_i^{(n)}(\vec{\rho}) \tag{7}$$

where $\vec{V}_i^{(n)}(\vec{\rho})$ is the vector basis function defined as

$$\vec{V}_i^{(n)}(\vec{\rho}) = \frac{\vec{\rho}_i^{(n)}}{2A^{(n)}} \tag{8}$$

and $I_i^{(n)}$ is the unknown current coefficient to be determined. In terms of the above-mentioned current basis functions, the EFIE (1) becomes

$$\hat{t}\cdot(k_c^2 + \nabla\nabla\cdot)\sum_n \int_{T^n} \overleftrightarrow{G}(\vec{r}\,|\,\vec{r}\,') \cdot \sum_{i=1}^{3} I_i^{(n)} \frac{\vec{\rho}_i^{(n)\prime}}{2A^{(n)}} ds_n' = 0 \tag{9}$$

To verify the validation of the vector basis representation for microstrip patch current, we may take the divergence of $\vec{J}_s^{(n)}(\vec{\rho})$, which is

$$\nabla\cdot \vec{J}_s^{(n)}(\vec{\rho}) = \sum_{i=1}^{3} \frac{I_i^{(n)}}{A^{(n)}} \tag{10}$$

In view of the equation of continuity, the charge density associated with the current (7) is constant over any triangular element. When the expansions (7) is substituted into the integral equation (1), the coefficients $I_i^{(n)}$ are constrained by the boundary conditions,

which require the continuity of the normal components of $\vec{J}_s$ across the edges shared by adjacent elements, or their vanishing at the boundary edges of the patch.

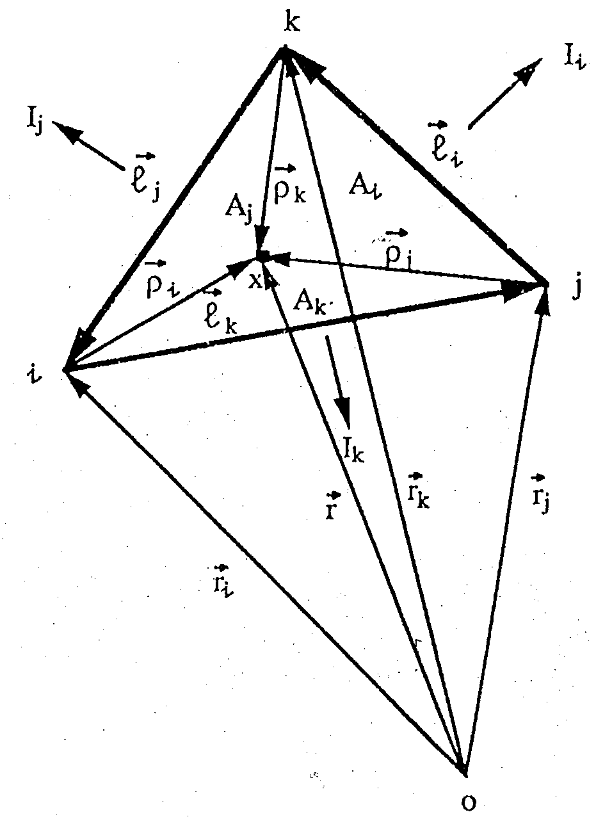

Fig.2. Local coordinates associated with a triangular element. The element number superscripts have been omitted.

B. Testing procedure and matrix assembly

In the moment method solution procedure, we need to test Eq.(9), which means to take a dot product of this equation with vector basis function $\vec{V}_\ell^{(m)}$ ($\ell = 1, 2, 3$), or equivalently, $\vec{\rho}_\ell^{(m)}$ ($\ell = 1, 2, 3$), and integrate the result over T^m

$$0 = \int_{T^m} \vec{V}_\ell^{(m)} \cdot \left[(k_c^2 + \nabla\nabla\cdot) \sum_{n=1}^{N} \int_{T^n} \overleftrightarrow{G}(\vec{r}\,|\,\vec{r}\,') \cdot \sum_{i=1}^{3} I_i^{(n)} \vec{V}_i^{(n)} ds_n' \right] ds_m$$

$$m = 1, 2, 3 \cdots \quad , N \tag{11}$$

For the sake of simplicity, we use a one-point rule to approximate the testing integrals. That is, to approximate $\vec{\rho}(r)$ and $\overleftrightarrow{G}(\vec{r}\,|\,\vec{r}')$ with the corresponding values $\vec{\rho}$ at the mth triangular centroid. Thus Eq.(11) becomes

$$0=\vec{\rho}_{c\ell}^{(m)}\cdot(k_c^2+\nabla\nabla\cdot)\sum_{n=1}^{N}\sum_{i=1}^{3}I_i^{(n)}\int_{T^n}\overleftrightarrow{G}(\vec{r}_c^{(m)}\,|\,\vec{r}')\cdot\vec{V}_i^{(n)}\,ds_n'$$

$$m=1,2,\cdots,N \qquad (12)$$

where the center of gravity of T^m is specified by the global position vector $\vec{r}_c^{(m)}$ and the local vector $\vec{\rho}_{c\ell}^{(m)}$.

Eq(12) are most conveniently evaluated by transforming from the global coordinate system to a local system of coordinate defined within T^n, which is the so called normalized area coordinates [11]. The integral transformation from Cartesian to normalized area coordinate for any function $g(\vec{r}')$ may be written in vector form as

$$\vec{r}'=\sum_{i=1}^{3}\vec{r}_i L_i'$$

where L_i' are subject to the constraint of (5), and

$$\int_{T^n}g(\vec{r}')\frac{d\,s_n}{2A^{(n)}}=\int_0^1\int_0^{1-L_j'}g(\vec{r}')\,dL_k'\,dL_j' \qquad (13)$$

As a result, we obtain

$$0=\sum_{n=1}^{N}\sum_{i=1}^{3}I_i^{(n)}\vec{\rho}_{c\ell}^{(m)}\cdot(k_c^2+\nabla\nabla\cdot)\int_0^1\int_0^{1-L_j'}\overleftrightarrow{G}(\vec{r}_c^{(m)}\,|\,\vec{r}')\cdot(\vec{\ell}_k'L_j'-\vec{\ell}_j^{(n)\prime}L_k')\,dL_k'\,dL_j' \qquad (14)$$

which leads to a system of linear equations

$$\sum_{n=1}^{N}\sum_{i=1}^{3}Z_{\ell i}^{(mn)}I_i^{(n)}=0\ ,\quad \ell=1,2,3 \qquad (15)$$

where the impedance $Z_{\ell i}^{(mn)}$ is given as

$$Z_{\ell i}^{(mn)}=\vec{\rho}_{c\ell}^{(m)}\cdot(k_c^2+\nabla\nabla\cdot)\int_0^1\int_0^{1-L_j'}\overleftrightarrow{G}(\vec{r}_c^{(m)}\,|\,\vec{r}')\cdot(\vec{\ell}_k'L_j'-\vec{\ell}_j^{(n)\prime}L_k')\,dL_k'\,dL_j' \qquad (16)$$

IV. Modeling of an example rectangular patch resonator

Consider a rectangular patch with dimension W_x in x direction and W_z in z direction, which can be partitioned into $2\cdot N_x\cdot N_z$ patches of isosceles-right-angled triangular shape, as illustrated in Fig.3. The relation between the quantities mentioned above is

$$W_x=h\cdot N_x\ ,\quad W_z=h\cdot N_z$$

where h is the length of isosceles-right-angled side.

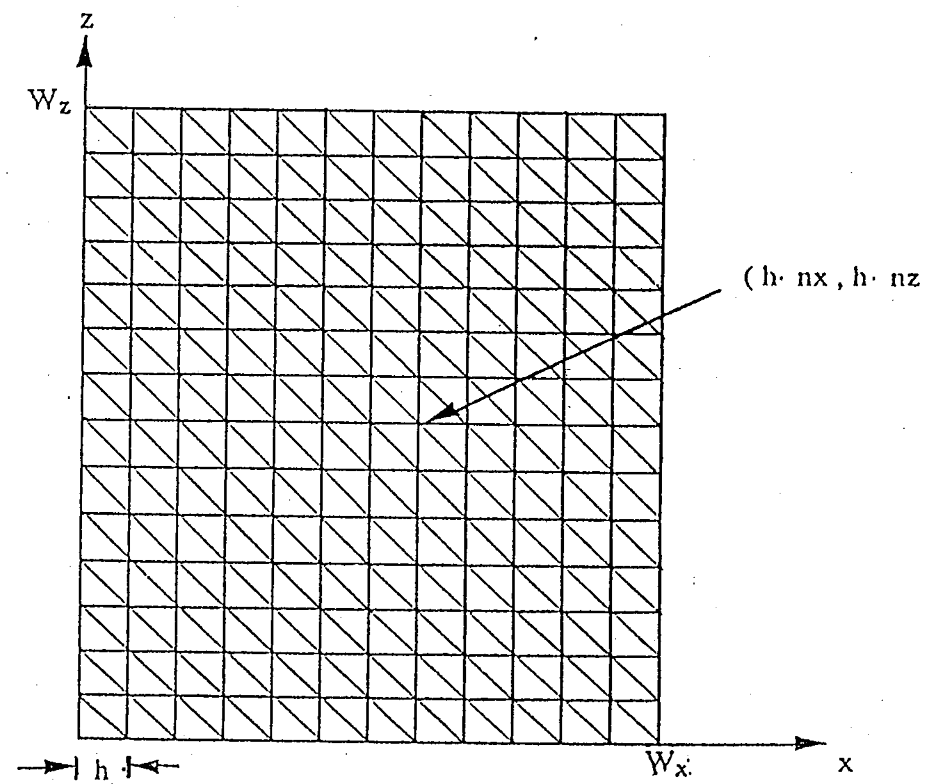

Fig.3. Modeling for a rectangular patch resonator

We classify these triangles into two groups. Group 1 indicates those triangles with rectangle in the down-and-left, group 2 indicates those triangles with rectangles in the up-and-right. Each node on the patch is assigned (x, z) in rectangular coordinate, with

$x=h\cdot nx\ ,\ z=h\cdot nz$

$nx_1=0,1,2,\cdots,(N_x-1)\ ,\ nz_1=0,1,2\cdots,(N_z-1)$

$nx_2=1,2,\cdots,N_x\ ,\ nz_2=1,2,\cdots,N_z$

The notation 1 means for group 1, 2 means for group 2. From now on, we just deduce the formula for group 1, since it follows the same procedure to do with the group 2.

In the nth source element, the position vector of three nodes $\vec{r}_i^{(n)}$, i = 1, 2, 3 and the three corresponding side length vectors $\vec{\ell}_i^{(n)\prime}$, i = 1, 2, 3 and the local vectors $\vec{\rho}_i^{(n)}$, i = 1, 2, 3 from nodes to any point within the element are as follow

$\vec{r}_{i1}^{(n)} = (h\cdot nx_1 , h\cdot nz_1)$

$\vec{r}_{j1}^{(n)} = (h\cdot nx_1 , h\cdot (nz_1+1))$

$\vec{r}_{k1}^{(n)} = (h\cdot (nx_1+1) , h\cdot nz_1)$

$\vec{\ell}_{i1}' = (-h , h)$

$\vec{\ell}_{j1}' = (0 , -h)$

$\vec{\ell}_{k1}' = (h , 0)$

$\vec{\rho}_{i1} = (h\cdot L_j' , h\cdot L_k')$

$\vec{\rho}_{j1} = (-h\cdot L_j' - h\cdot L_k' , h\cdot L_k')$

$\vec{\rho}_{k1} = (h\cdot L_j' , -h\cdot L_j' - h\cdot L_i')$

For an isosceles-right-angled triangle with waist length h, as depicted in Fig.4, the point i, j, k, c is the three nodes and the centroid, we can calculate to get

$\overline{ic} = 0.4142136h$

$\overline{ji} = \overline{ki} = 0.7653669h$

$\vec{\rho}_{ci} = (0.29289h , 0.29289h)$

$\vec{\rho}_{cj} = (-0.70711h , 0.29289h)$

$\vec{\rho}_{ck} = (0.29289h , -0.70711h)$

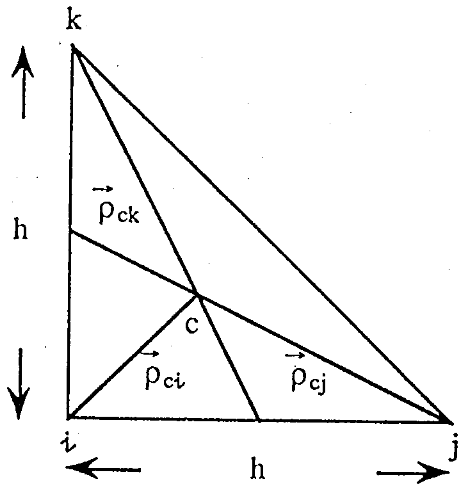

Fig.4. An isosceles-right-angled triangle with waist length h.

The position vector of the centroid point can be expressed as follows

$\vec{r}_{c1}^{(m)} = (h\cdot (mx_1+0.29289) , h\cdot (mz_1+0.29289))$

Now we have calculated all the notations shown in Eq.(16), and we substitute them into Eq.(16). Since the operator $(k_c^2 + \nabla\nabla\cdot) \overleftrightarrow{G}(\vec{r}_c^{(m)}|\vec{r}')$ and the double surface parameter integral is mutually independent, we assume the interchange of the two is valid. Next, we evaluate $(k_c^2 + \nabla\nabla\cdot) \overleftrightarrow{G}(\vec{r}_c^{(m)}|\vec{r}')$, and for convenience, we define $\overleftrightarrow{q}(\xi,\zeta)$ and take a reasonable assumption that both y and y' approach zero as follows

$$(k_c^2+\nabla\nabla\cdot) \cdot \overleftrightarrow{G}(\vec{r}_c^{(m)}|\vec{r}') = \int_\xi \int_\zeta (k_c^2+\nabla\nabla\cdot) \overleftrightarrow{g}(\vec{r}_c^{(m)}|\vec{r}')\, e^{j\vec{\lambda}\cdot(\vec{r}-\vec{r}')}\, d\zeta\, d\xi$$

$$= \int_\xi \int_\zeta \overleftrightarrow{q}(\xi,\zeta)\, e^{j\vec{\lambda}\cdot(\vec{r}-\vec{r}')}\, d\zeta\, d\xi \qquad (17)$$

where $\overleftrightarrow{q}(\xi,\zeta)$ is in dyadic form, and

$$q_{xx} = q_{zx} = (k_c^2-\xi^2)\cdot\frac{(1+R_t)}{2\cdot p_c} + \xi^2\,\frac{C}{2}$$

$$q_{xz} = q_{zz} = (-\xi\zeta)\cdot\frac{(1 + R_t - 2\cdot p_c)}{2\cdot p_c}$$

Eq.(16) now becomes

$$Z_{\ell i}^{(mn)} = \vec{\rho}_{c\ell}^{(m)} \cdot \int_\xi \int_\zeta \int_0^1 \int_0^{1-L_j'} \overleftrightarrow{q}(\xi,\zeta)\, e^{j\vec{\lambda}\cdot(\vec{r}-\vec{r}')} (\vec{\ell}_k^{(n)\prime} L_j' - \vec{\ell}_j^{(n)\prime} L_k')\, dL_k'\, dL_j'\, d\zeta\, d\xi \qquad (18)$$

Since $\vec{\rho}_{c\ell}^{(m)}$ is not a function of integration variables L_i', L_j' and L_k'; we can perform the calculation of $\vec{\rho}_{c\ell}^{(m)}\cdot$ before doing the surface integral $\int_\xi \int_\zeta ( )\, d\xi d\zeta$. The double integral on complex variable ξ, ζ will be computed numerically.

For numerical solution, we have to find out resonant frequencies which render the determinant of the impedance matrix vanish. The resonant frequencies are then substituted back to the matrix equation for solving the unknown current distributions fir each mode. A Fortran program is under development for the computation work; numerical results for some example structures are expected to be published in a coming paper.

V. Conclusion

In this work, a powerful, full-wave technique based on a rigorous spectral domain formulation of electric field integral equation is presented for the analysis of microstrip resonators of arbitrary shape. A special set of subdomain-type vector triangular basis functions is utilized in the MOM procedure to represent the unknown patch surface current. The selected vector basis functions are seen to be powerful in modeling arbitrary-shape patch surfaces. The approach developed in this research will be valided by using it to solve patch currents of simple, regular shapes. Work along this line is under process, and results are expected to be published in a coming papers.

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