Frequency Scaling Implementation of Adaptive Analog Filter Design by State Feedback Approach

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ABSTRACT

This research proposes a practical method to implement frequency scaling of adaptive analog filters. Frequency scaling is currently well known skill to transfer a low-pass prototype filter into another low-pass filter with different cut-off frequency. It is a mathematical methodology to have the passband change. Here, state feedback approach is used to shift poles of the original low-pass prototype filter to the desired poles location so that a new filter that has the desired passband and the desired cut-off frequency can be obtained. This approach also makes filters potentially achieve adaptive performances. Frequency scaled filters and state feedback controlled filters are also compared to show the feasibility of this approach in this paper.

Keywords: frequency scaling, adaptive, analog, filter, state feedback

利用狀態回授法實踐適應性類比過濾器之頻率比率化調變

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摘　　要

這篇研究提出利用狀態回授法實踐適應性類比過濾器之頻率比率化調變，頻率比率化調變是現今皆知之技術用以轉換一個原形之低通濾器成為一個不同截止頻率和不同通帶之低通濾波器．此為一數學上改變過濾器通帶之方法．這文章利用狀態回授法把原形之低通濾器之極點移到所想要之濾波器極點，使此原形之低通濾器轉變為不同截止頻率和不同通帶之低通濾波器．此法將使濾波器具有適應性之能力．

關鍵字：頻率比率化，適應性，類比，過濾器，狀態回授

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I. INTRODUCTION

Frequency scaling is one of the major concepts to design low-pass filters. It will affect further design on high-pass filters, band-pass filters or band-reject filters [1]. This process adjusts passband of the prototype low-pass filters by changing cut-off frequency. In the practical implementation of changing cut-off frequency, the filter circuit may require switching different values of electronic devices [2]. If filter design can make filters have different passband without switching any electronic devices, this contribution would be huge in the filter design field [3,4].

If filters can be adjustable due to different environments or different desired working conditions. It would reduce cost of filters and size of products; therefore, to make filters adaptive become a major issue to concern with the current filter design. For example, in communication systems, simplifying the filter circuit is very important. Cost and size could affect sales. Communication systems are expected to experience many different kinds of environments and working conditions, such as temperature change, shaking environment and the desired passband. These all require changing the characteristics of filters. So, adaptive control for filters becomes a major concept to overcome these problems. State feed-back control would be a good method to implement this task.

From the studying of analog filter design, it clearly shows that every different kind of filter has its own poles [5,6]. The poles are like fingerprints to human in filters; therefore, if we can appropriately utilize this characteristic, new intelligent filters can be designed.

State feedback is applied to achieve the goal of building such intelligent filters in here [6]. State feedback is known to use in close-loop control in state models [7]. In the other words, modern control theory is used to have a close-loop control in filters [8]. By this approach, characteristics of filters can be changed. In state feedback control, it is known that if state variables are not available, observers are needed. In the practical filter design, a passive filter is usually a RLC circuit, state variables of filters are easy measured and obtained; therefore, consideration of observers is unnecessary.

II. APPROACH

A block diagram shown in Fig. 1 is sketched to show the approach of the adaptive filter design. The overall process of design can be probably divided into three steps. The first step is to find the state model of the original low-pass prototype filter, $H_{LPP}(S)$ that represents all types of filters such as butterworth filters and chebyshev filters . The second step is to find the poles of frequency transformed state models such as frequency scaled filters, band-pass filters and high-pass filters. The final step is to use the poles found in the second step as desired poles for the state model of $H_{LPP}(S)$; therefore, state feedback gains used to transfer a low-pass prototype filter to another type of filter can be found.

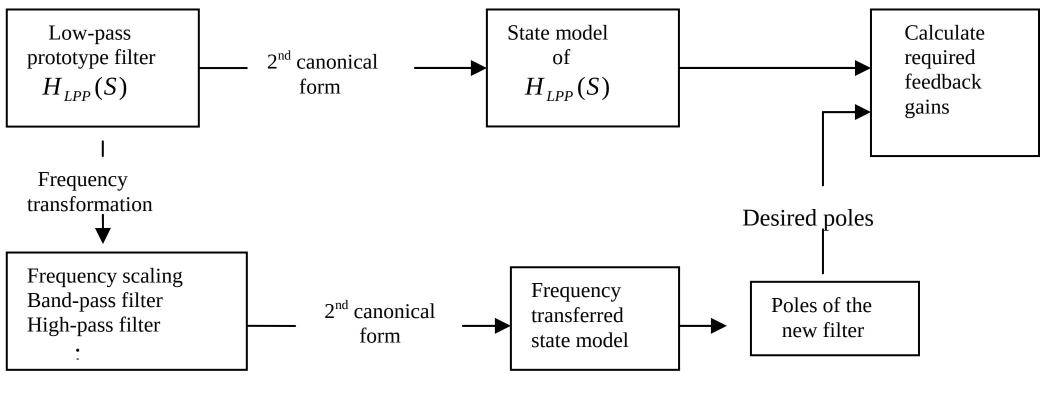


Fig. 1. The design flow of adaptive filters design.

III. MAIN RESULTS

Consider a general filter function $H_{LPP}(S)$ that denotes the low-pass prototype transfer function and can be presented by a polynomial in S over a polynomial in S

$$H_{LPP}(S) = \frac{\sum_{k=0}^{M} b_k S^k}{\sum_{k=0}^{N} a_k S^k} \qquad (1)$$

where $M = N =$ the highest order of numerator or denominator. a_k and b_k are constant parameters of denominator and numerator respectively.

The reason why $M = N$ is defined, because it helps to simplify sketching the block diagram of filters by second canonical form in the further step. A frequency scaled filter transfer function $H^{fs}(S)$ can be obtained by frequency transformation with a scaling factor k_f that is real and positive [9].

$$H^{fs}(S) = H_{LPP}(S)|_{S \to S/k_f} = \frac{\sum_{k=0}^{M} b_k (S/k_f)^k}{\sum_{k=0}^{N} a_k (S/k_f)^k} \qquad (2)$$

Let $b_k^{\ fs} = b_k/(k_f)^k$ and $a_k^{\ fs} = a_k/(k_f)^k$. So,

$$H^{fs}(S) = \frac{\sum_{k=0}^{M} b_k^{\ fs} S^k}{\sum_{k=0}^{N} a_k^{\ fs} S^k} \qquad (3)$$

Now, Apply second canonical form and define a variable w. Eq.(1) can be presented as

$$\frac{Y(S)}{U(S)} = \frac{(\sum_{k=0}^{M} b_k S^k)w}{(\sum_{k=0}^{N} a_k S^k)w} \qquad (4)$$

where $Y(S) = (\sum_{k=0}^{M} b_k S^k)w;\ U(S) = (\sum_{k=0}^{N} a_k S^k)w.$

In time-domain,

$$y = b_M(w)^{(M)} + b_{M-1}(w)^{(M-1)} + .\,.......+ b_1(w)' + b_0 w \qquad (5)$$

$$u = a_N(w)^{(N)} + a_{N-1}(w)^{(N-1)} + .\,.......+ a_1(w)' + a_0 w \qquad (6)$$

where the superscript (M) and (N) denote derivative order of w

Eq.(6) can be revised as

$$(w)^N = -\frac{a_{N-1}}{a_N}(w)^{(N-1)} - .... - \frac{a_1}{a_N}(w)' - \frac{a_0}{a_N}(w) + \frac{u}{a_N} \qquad (7)$$

Based on Eq.(5) and Eq.(7), a block diagram of the low-pass prototype filter, $H_{LPP}(S)$, can be sketched as Fig. 2.

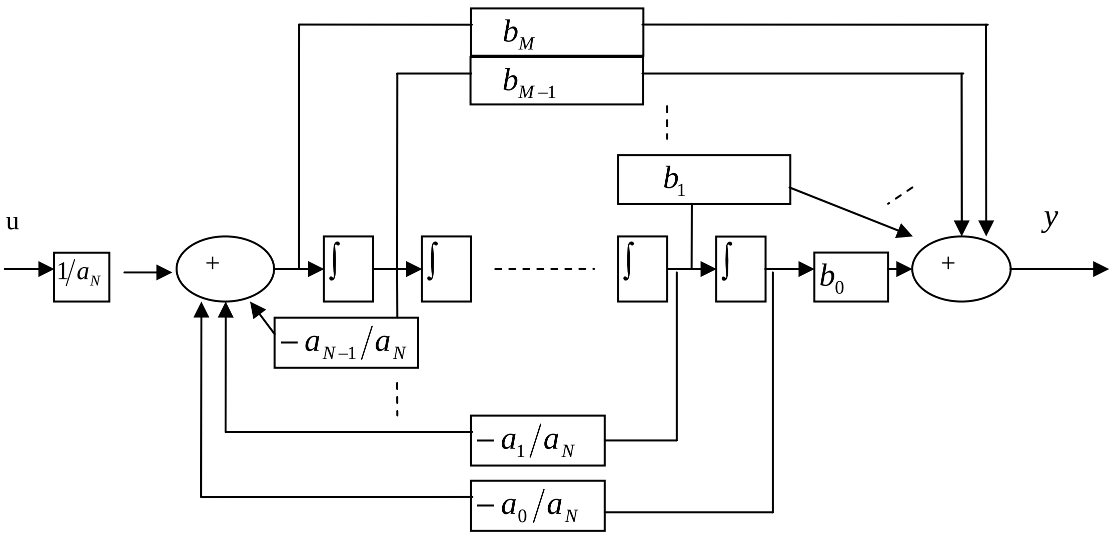


Fig. 2. The block diagram of $H_{LPP}(S)$.

A state model obtained from Fig. 2 is shown as

$$\begin{cases} \dot{x}_1 = x_2 \\ \quad \vdots \\ (\dot{x}_p) = -\frac{a_0}{a_p}(x_1) - \frac{a_1}{a_p}(x_2) - .... - \frac{a_{p-1}}{a_p}(x_p) + \frac{u}{a_p} \end{cases} \qquad (8a)$$

$$y = b_0 x_1 + b_1 x_2 + \dots\dots + b_{M-1}(x_M) + b_M(\dot{x}_M) \quad (8b)$$

Because of $M = N$, $x_M = x_N$. For simplification, define another state variable x_p to represent x_M, x_N at the same time with $p = 1 \sim N$, $p = 1 \sim M$ and $M = N = P$. Therefore,

$$y = b_0 x_1 + b_1 x_2 + \dots\dots + b_{p-1}(x_p) + b_p(\dot{x}_p) \quad (9)$$

where

$$(\dot{x}_p) = -{}^{a_0}\!/_{a_p}(x_1) - {}^{a_1}\!/_{a_p}(x_2) - \dots\dots - {}^{a_{P-1}}\!/_{a_p}(x_p) + {}^{u}\!/_{a_p}$$

then,

$$\begin{aligned} y &= b_0 x_1 + b_1 x_2 + \dots\dots + b_{p-1}(x_p) + \\ &\quad b_p[-{}^{a_0}\!/_{a_p}(x_1) - {}^{a_1}\!/_{a_p}(x_2) - \dots\dots - {}^{a_{p-1}}\!/_{a_p}(x_p) + {}^{u}\!/_{a_p}] \\ &= (b_0 - \frac{b_p a_0}{a_p})x_1 + (b_1 - \frac{b_p a_1}{a_p})x_2 + \dots\dots \\ &\quad + (b_{p-1} - \frac{b_p a_{p-1}}{a_p})x_p + \frac{b_p}{a_p}u \end{aligned} \quad (10)$$

Now, use a standard state model to represent the state equation and the output equation derived from Fig. 2.

$$\begin{cases} \dot{x} = Ax + Bu & (11a) \\ y = Cx + Du & (11b) \end{cases}$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ : \\ x_p \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ : & : & : & \dots & : \\ 0 & 0 & 0 & : & 1 \\ \frac{-a_0}{a_p} & \frac{-a_1}{a_p} & .. & \dots & \frac{-a_{p-1}}{a_p} \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ : \\ 0 \\ \frac{1}{a_p} \end{bmatrix}$$

$$C = \begin{bmatrix} (b_0 - \frac{b_p a_0}{a_p}) & (b_1 - \frac{b_p a_1}{a_p}) & .. & .. & (b_{p-1} - \frac{b_p a_{p-1}}{a_p}) \end{bmatrix}$$

$$D = \begin{bmatrix} \frac{b_p}{a_p} \end{bmatrix}$$

Next, for the state model of frequency scaled filters $H^{fs}(S)$. The same process finding state model of $H_{LPP}(S)$ by second canonical form can be applied too. So, in time-domain, Eq.(3) can be also shown as Eq.(12) and Eq.(13) and a block diagram can be sketched as Fig. 3.

$$y = b_M{}^{fs}(w)^{(M)} + b_{M-1}{}^{fs}(w)^{(M-1)} + \dots\dots + b_1{}^{fs}(w)' + b_0{}^{fs}w \quad (12)$$

$$u = a_N{}^{fs}(w)^{(N)} + a_{N-1}{}^{fs}(w)^{(N-1)} + \dots\dots + a_1{}^{fs}(w)' + a_0{}^{fs}w \quad (13)$$

where the superscript (M) and (N) denote derivative order of w.

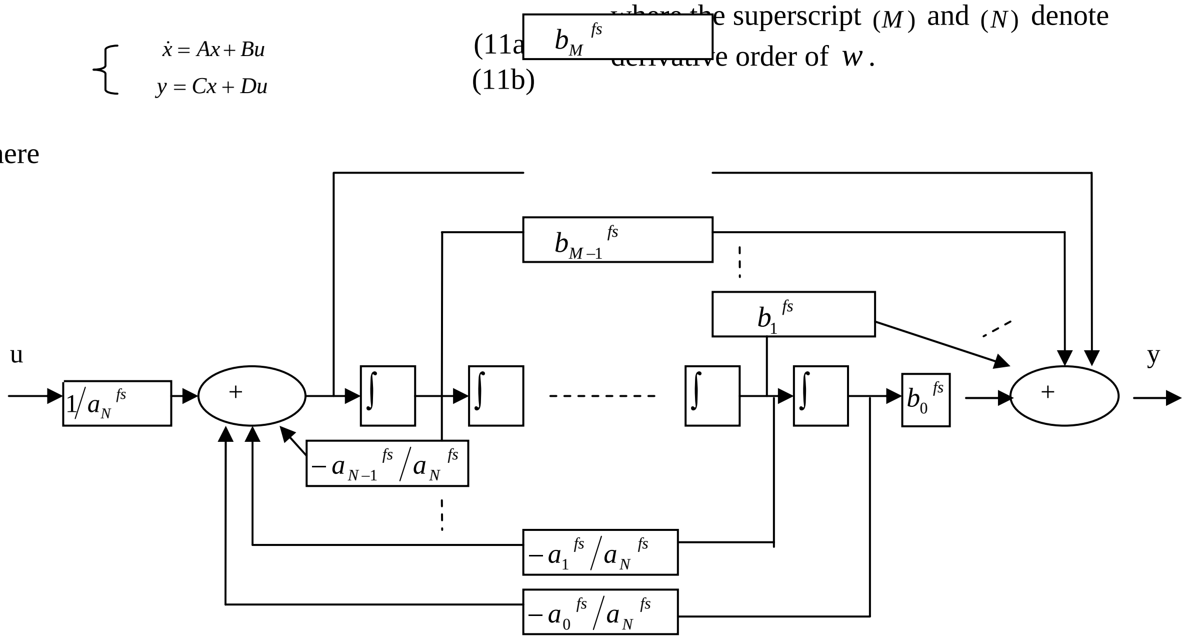


Fig. 3. The block diagram of $H^{fs}(S)$.

A state model obtained from Fig. 3 is shown as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \quad : \\ (\dot{x}_N) = -{}^{a_0{}^{fs}}\!/_{a_N{}^{fs}}(x_1) - {}^{a_1{}^{fs}}\!/_{a_N{}^{fs}}(x_2) - \dots\dots - {}^{a_{N-1}{}^{fs}}\!/_{a_N{}^{fs}}(x_N) + {}^{u}\!/_{a_N{}^{fs}} \end{cases} \quad (14a)$$

$$y = b_0{}^{fs}x_1 + b_1{}^{fs}x_2 + \dots\dots + b_{M-1}{}^{fs}(x_M) + b_M{}^{fs}(\dot{x}_M) \quad (14b)$$

Same here, define another state variable x_p to represent x_M, x_N at the same time with $p = 1 \sim N$, $p = 1 \sim M$ and $M = N = P$ for simplification. Therefore,

$$y = b_0{}^{fs} x_1 + b_1{}^{fs} x_2 + ... + b_{p-1}{}^{fs}(x_p) + b_p{}^{fs}(\dot{x}_p) \quad (15)$$

where

$$(\dot{x}_p) = -a_0{}^{fs}/a_p{}^{fs}(x_1) - a_1{}^{fs}/a_p{}^{fs}(x_2) - .... - a_{p-1}{}^{fs}/a_p{}^{fs}(x_p) + u/a_p{}^{fs} \quad (16)$$

then,

$$y = b_0{}^{fs} x_1 + b_1{}^{fs} x_2 + ... + b_{p-1}{}^{fs}(x_p) + b_p{}^{fs}[-a_0{}^{fs}/a_p{}^{fs}(x_1) - a_1{}^{fs}/a_p{}^{fs}(x_2) - .... - a_{p-1}{}^{fs}/a_p{}^{fs}(x_p) + u/a_p{}^{fs}] \quad (17)$$

$$= (b_0{}^{fs} - \frac{b_p{}^{fs} a_0{}^{fs}}{a_p{}^{fs}})x_1 + (b_1{}^{fs} - \frac{b_p{}^{fs} a_1{}^{fs}}{a_p{}^{fs}})x_2 + ... + (b_{p-1}{}^{fs} - \frac{b_p{}^{fs} a_{p-1}{}^{fs}}{a_p{}^{fs}})x_p + \frac{b_p{}^{fs}}{a_p{}^{fs}}u \quad (18)$$

Now, use a standard state model to represent the state equation and the output equation derived from Fig. 3.

$$\begin{cases} \dot{x} = A^{fs} x + B^{fs} u & (19a) \\ y = C^{fs} x + D^{fs} u & (19b) \end{cases}$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ : \\ x_p \end{bmatrix}, \qquad A^{fs} = \begin{bmatrix} 0 & 1 & 0 & .... & 0 \\ 0 & 0 & 1 & .... & 0 \\ : & : & : & .... & : \\ 0 & 0 & 0 & : & 1 \\ \frac{-a_0{}^{fs}}{a_p{}^{fs}} & \frac{-a_1{}^{fs}}{a_p{}^{fs}} & .. & .... & \frac{-a_{p-1}{}^{fs}}{a_p{}^{fs}} \end{bmatrix}$$

$$B^{fs} = \begin{bmatrix} 0 \\ : \\ 0 \\ \frac{1}{a_p{}^{fs}} \end{bmatrix}$$

$$C^{fs} = \begin{bmatrix} (b_0{}^{fs} - \frac{b_p{}^{fs} a_0{}^{fs}}{a_p{}^{fs}}) & (b_1{}^{fs} - \frac{b_p{}^{fs} a_1{}^{fs}}{a_p{}^{fs}}) & .. & .. & (b_{p-1}{}^{fs} - \frac{b_p{}^{fs} a_{p-1}{}^{fs}}{a_p{}^{fs}}) \end{bmatrix}$$

$$D^{fs} = \begin{bmatrix} \frac{b_p{}^{fs}}{a_p{}^{fs}} \end{bmatrix}$$

Now, in order to frequency scale the low-pass prototype filter, the desired poles of $H_{LPP}(S)$ state model that are the eigenvalues of the matrix A^{fs} need to be found first [9]. Therefore,

$$P_d = eig(A^{fs}) \quad (20)$$

Define a state feedback gain $-G$ for Eq.(11), $H_{LPP}(S)$ state model. Then, the state feedback can be wrote as

$$u = -Gx \quad (21)$$

Next, Eq.(11a) can be revised as

$$\dot{x} = Ax - BGx \quad (22)$$

$$= (A - BG)x \quad (23)$$

Let $eig(A - BG) = eig(A^{fs})$ (24)

The state feedback gain $-G$ can be obtained by comparing the characteristic polynomials of matrix $(A - BG)$ and (A^{fs}).

$$\det(SI - (A - BG)) = \det(SI - A^{fs}) \quad (25)$$

where I is an identity matrix.

$-G$ will be the gain that transfer the low-pass prototype filter into the frequency scaled new filter by state feedback approach. State-feedback only changes the pole of the system. In the other words, feedback gains only affect the state equation of the system. The output equation of the close-loop controlled system should use the output equation of the frequency transformed state model, because the block diagram of close-loop controlled filter block diagram should has same structure as the block diagram of frequency transformed state model.

Such a low-pass to low-pass transformation by state feedback control, the features of filters can be summarized as:

A. Poles and Zeros

Since k_f is real and positive, the magnitude of the poles and zeros are scaled but not the phase angles. A pole/zero plot of $H^{fs}(S)$ will be identical to that of $H_{LPP}(S)$ except that the axes will be scaled by k_f.

B. Magnitude Frequency Response and Phase Response

The magnitude frequency response and phase response maybe summarized as follows:

$$|H^{fs}(jw)| = |H_{LPP}(jw/k_f)| \quad (26)$$

and, $\angle H^{fs}(jw) = \angle H_{LPP}(jw/k_f)$ (27)

Therefore, plots of the transformed (frequency-scaled) magnitude frequency response, (26), and the phase response, (27), are identical to those obtained from the low-pass prototype except for the frequency axes by k_f.

C. Phase Delay and Group Delay

Phase delay and group delay may be summarized as follows:

$$t_{pd}^{fs}(w) = t_{pd}^{LPP}(w/k_f)/k_f \quad (28)$$

and,

$$t_{pd}^{fs}(w) = t_{gd}^{LPP}(w/k_f)/k_f \quad (29)$$

Therefore, plots of the transformed (frequency-scaled) phase delay (28), and group delay (29), not only have the frequency axes scaled by k_f, but the amplitude axes and scaled by $1/k_f$.

D. Time-Domain Response

The unite impulse response may be summarized as follow:

$$h^{fs}(t) = k_f h_{LPP}(k_f t) \quad (30)$$

Note that the transformed (frequency-scaled) unit impulse response, as shown in (30), is a time—scaled version of the prototype unit impulse response and is also amplitude scaled by k_f. If $k_f > 1$, then $h^{fs}(t)$ will be greater in amplitude and time-compressed, compared to $h_{LPP}(t)$.

The unit step response may be summarized as follow:

$$h_u^{fs}(t) = h_u^{LPP}(k_f t)$$

Note that the transformed unit step response, as shown in Eq.(31), is a time scaled version of the prototype unit step response with no corresponding amplitude.

IV. ILLUSTRATIONS

A 3th order Butterwoth low-pass prototype transfer function

$$H_{LPP}(S) = \frac{1}{S^3 + 2S^2 + 2S + 1} \quad \text{with } Wc = 1 \quad (32)$$

The frequency response and the pole-zero plot are shown in Fig. 4.

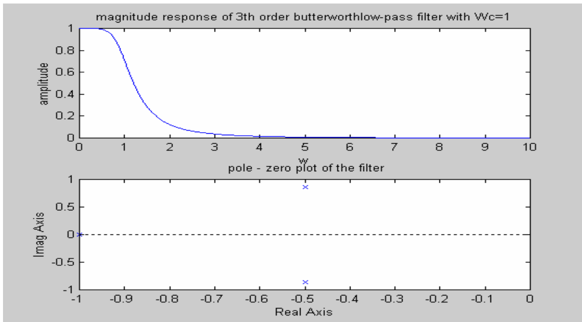


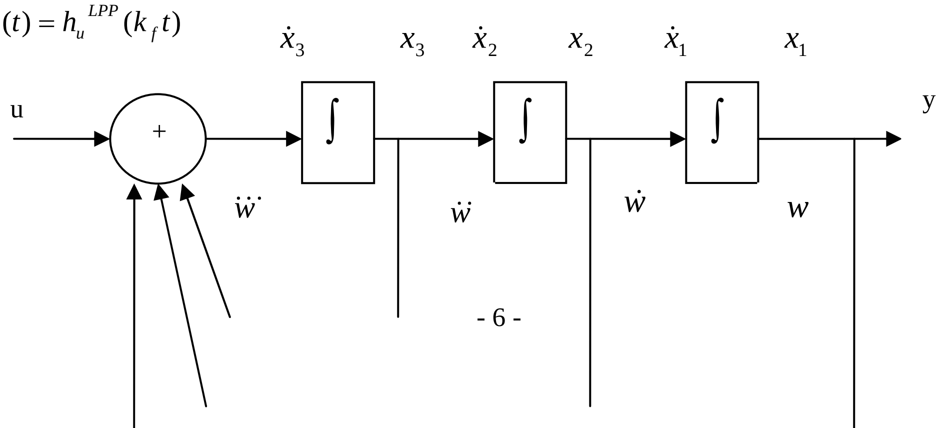
Fig. 4 The frequency response and the pole-zero plot.

If it is desired that the cut-off frequency, $Wc = 100$. First of all, in order to find the state model of $H_{LLP}(S)$. The block diagram of the filter needs to be drawn by the 2nd canonical from as Fig. 5.

Therefore, the state model is shown as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad (33a)$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (33b)$$



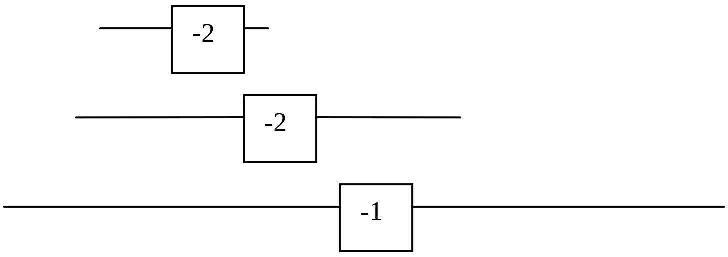


Fig. 5. The block diagram of $H_{LPP}(S)$.

Next, for the state model of Wc =100,

$$H^{fs}(S) = H_{LPP}(S)|_{S \to S/100} = \frac{10^6}{S^3 + 2\times10^2 S^2 + 2\times10^4 S + 10^6} \quad (34)$$

The frequency response and the pole-zero plot of $H^{fs}(S)|_{Wc=100}$ are shown in Fig. 6.

And the block diagram of $H^{fs}(S)|_{Wc=100}$ is shown as Fig. 7. Therefore, The state model of $H^{fs}(S)|_{Wc=100}$ is shown as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -1\times10^6 & -20000 & -200\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\times10^6\end{bmatrix}u \quad (35a)$$

$$y = \begin{bmatrix}1\times10^6 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} \quad (35b)$$

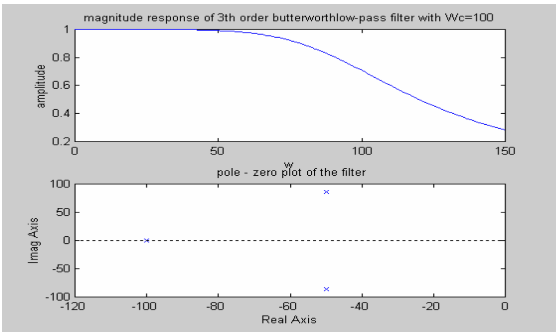
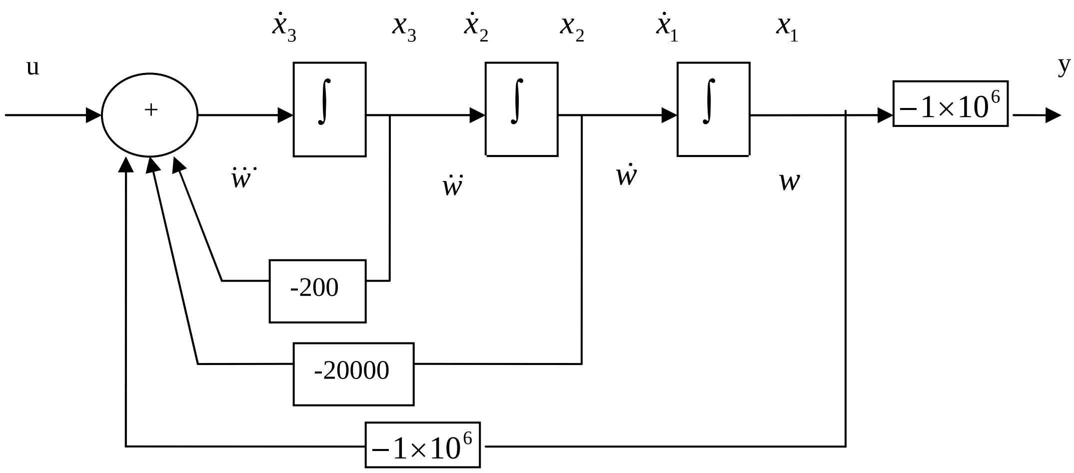


Fig 6. The frequency response and the pole-zero plot of $H^{fs}(S)|_{Wc=100}$.



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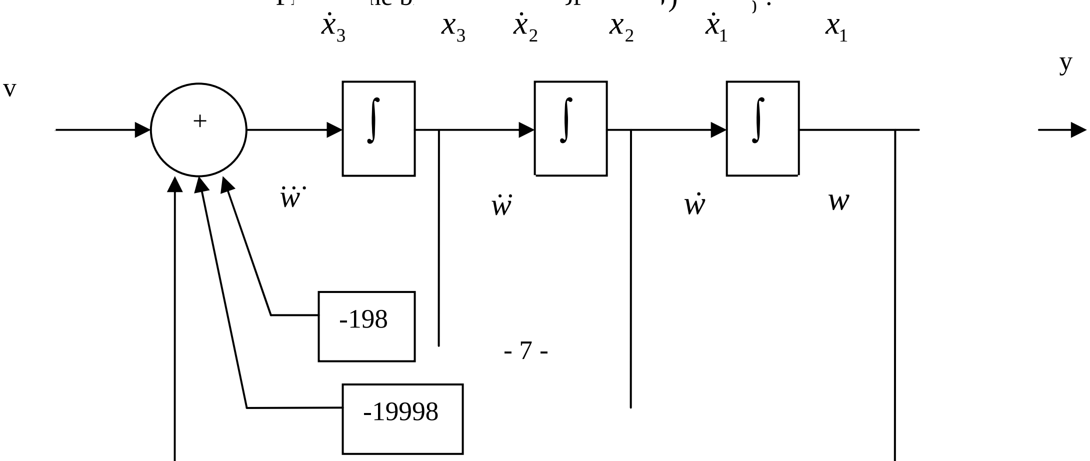

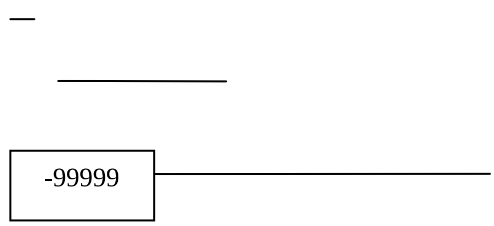

Fig. 8. The block diagram of the close-loop controlled filter with preliminary feedback $u = -Gx + v$.

The poles of $H^{fs}(S)\ |_{Wc=100}$ are -100, $-50 \pm 86.6i$ that will be the desired pole of close-loop control on the $H_{LLP}(S)$ state model. Therefore, the state feedback gains that can transfer $H_{LLP}(S)$ to $H^{fs}(S)\ |_{Wc=100}$ is $G = [99999 \quad 19998 \quad 198]$ and the close-loop controlled filter model

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -100000 & -20000 & -200 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (36a)$$

-1×10^6

This output equation uses the same numerator polynomial as Eq.(34). In the other words, the block diagram drawn by the second canonical form of this close-loop controlled filter shown in Fig. 8 will have same structure as Fig. 7.

$$y = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (36b)$$

The frequency response and the pole-zero plot of this close-loop controlled filter are show as below.

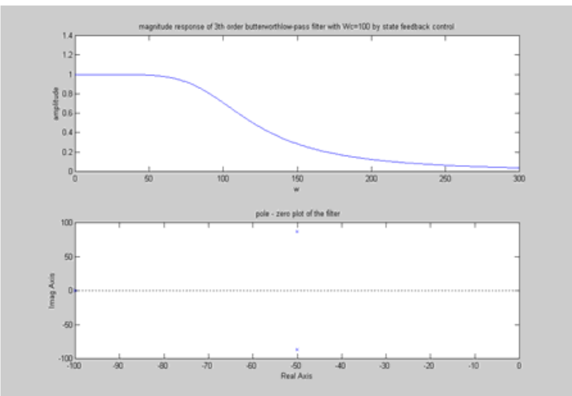

Fig. 9. The frequency response and the pole-zero plot of this close-loop controlled filter.

Now, we can compare the theoretical frequency scaled filter in Fig. 6 and practically implementing state feedback control in Fig. 9. These two low-pass filters having Wc = 100 are almost identical. This shows the feasibility of achieving adaptive filters by state feedback approach.

VII. CONCLUSIONS

This paper is primary research of implementing adaptive analog filter design by state feedback approach. It provides a new concept to implement frequency scaling of filter design. This technique is used in not only low-pass to low-pass transformation but also low-pass to high pass transformation, low-pass to band-pass transformation, and low-pass to band-reject transformation. This method does not restrict using in any type of filters. This research opens a new door for adaptive filter design field. Based on one filter circuit, low-pass filters, high pass filters, and band-pass filters will all be achieved without changing circuit design or switching any electronic devices.

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