工業教育與技術學刊
第三十期　民國九十五年　第 79～90 頁
Journal of Industrial Education and Technology
Vol. 30. December. 2006, PP. 79～90

分散驅動式殊異擾動電腦控制系統之強健性及可靠性探討

姚 凱 超

國立彰化師範大學
工業教育與技術學系

摘　要

在這篇研究中，對於電腦控制的分散驅動式殊異擾動系統之強健性及可靠性加以研究和討論。因為此系統是使用減階控制器控制，因此，系統的強健性必須被加以考量。在研究中也發展一種強健控制的測試方法，系統亦可由此法找出強健控制之範圍。可靠性在於分散式殊異擾動系統是指每個控制端的控制器可以穩定受控系統，且使得受控系統可承受任意控制端的失效。這篇研究的結果顯現出只要系統符合一特定架構， 且具有一定的條件的話，此系統將會是一個可靠性的控制系統。

關鍵字：強健性，可靠性，殊異擾動。

Journal of Industrial Education and Technology
Vol. 30. December. 2006, PP. 79～90

Investigation of Robustness and Reliability in Computer Control of Decentralized Singularly-perturbed Actuator Systems

Kai-chao Yao

National Chang-hua University of Education
Department of Industrial Education and Technology

ABSTRACT

Robustness investigation and reliability investigation are discussed and investigated in his research for computer control of decentralized singularly-perturbed systems. Due to the systems are controlled by using reduced-order control scheme, robustness of the system should be necessarily concerned. A procedure of a robust control test will be developed to find the robustness bound of the systems. A reliability goal for a decentralized system is the stabilization of the plant by a controller in each control channel, such that the system can tolerate control channel failures. The results of the research show the system that has a certain structure with certain conditions will be a reliable control system.

Key Words: Robustness, Reliability, Singularly-perturbed.

I. INTRODUCTION

This paper is an extension research of [1]. In [1], Digital Observer-based Controller Design of Decentralized Actuator Systems is developed.

Most of the engineering analysis and design tools rely on a precise mathematical description of the physical problem. Nevertheless, in most of the practical problems we can not specify such model (unstructured uncertainty) or the value of its parameters (structured uncertainty). In addition to the description and propagation of the uncertainty, the output subjected to optimization is now parameterized by the design variables. Considerable effort has been devoted to the development and implementation of viable numerical strategies. While these tools are the only alternative for studying large scale problems, their nature often precludes a deeper understanding of the problem and its solution. This paper studies the robustness and reliability of decentralized singularly-perturbed systems with structured uncertainty by multiple control technique. This allows to insure the controllers to have such systems present the desired performances.

Robust control and reliable control are two of the major concerns for the designs in decentralized large-scale systems. Here, the control scheme of decentralized singularly-perturbed systems will be investigated for robust control and reliable control.

The state model will be always an inaccurate representation of the actual physical system because of parameter changes, unmodeled dynamics, unmodeled time delays, changes in equilibrium point, sensor noise, and unpredicted disturbance inputs [3]. A robust control system exhibits the desired performance despite the presence of significant uncertainty. In this research, the unmodel dynamics will be the major uncertainty considered, because decentralized large-scale systems are reduced to lower-order systems by neglecting the fast state variables that only affect the system responses in the very initial time period. Therefore, in this research, robust control is defined as that the desired performances are still exiting after applying the decentralized reduced-order controller in singularly-perturbed full-order systems. The bound of robust control is expected to be found in the investigation.

Due to increasing complexity of modern technological process, reliability of control has become an essential requirement in the design of large-scale systems [2], [4]-[8]. In a decentralized high-dimension system, if a local controller breaks down, it is entirely feasible that the whole system may do the same. Replacement of a faulty controller by a standby, or disconnection of the corresponding subsystem for the purpose of preventing the system breakdown may be either impossible or undesirable due to the design constraints. A reliability goal for a decentralized system is the stabilization of the plant by a controller in each control channel [2], [4].

In decentralized control systems, basically there are two types of control schemes. The control scheme of the type-one, the controller of each channel only controls partial overall state variables [4]. The control scheme of the type-two, the controller of each channel can control overall state variables [2]. The second type of decentralized control scheme is considered as a potential reliable control scheme, because of the structure. In the type-two system, if the controller of each channel is able to stabilize the system, the system possesses reliable control properties. Due to the way of designs, the decentralized control scheme that is concerned in this research belongs to the type-two system.

II. SYSTEM DESCRIPTION

The mathematical model of the system is shown as below:

$$\dot{x} = A_{00}x + \sum_{i=1}^{m} A_{0i} z_i \tag{1a}$$

$$\varepsilon\dot{z}_i = A_{i0}x + A_{ii}z_i + B_i u_i \tag{1b}$$

$$y_i = C_i z_i \tag{1c}$$

where i=1~m

The system is a linear time-invariant decentralized singularly-perturbed actuator system which has n-order and m independent inputs or m sub-systems. $x \in R^S$ and $z \in R^F$ are the slow and the fast state variables respectively; each sub-system z_i has its own order. $u_i \in R^{n_{1i}}$ and $y_i \in R^{n_{2i}}$ are the input vector of the i-th subsystem and the output vector of the i-th subsystem respectively. A_{00}, A_{0i}, A_{i0}, A_{ii} , B_i and C_i are constant matrices with appropriate dimensions with i=1~m.

After apply singularly-perturbed methods and analog to digital transformation technique, if we have control from the subsystem one, the system can be shown as below [1] and the structure is shown in Figure 1.

$$x_r(k+1) = [\Phi_N + \Gamma_1 K_1(k)]x_r(k) + GN(k) \tag{2a}$$

where $\Phi_N = (\Phi + \Gamma_2 K_2 + \Gamma_3 K_3 + \ldots.\Gamma_m K_m)$

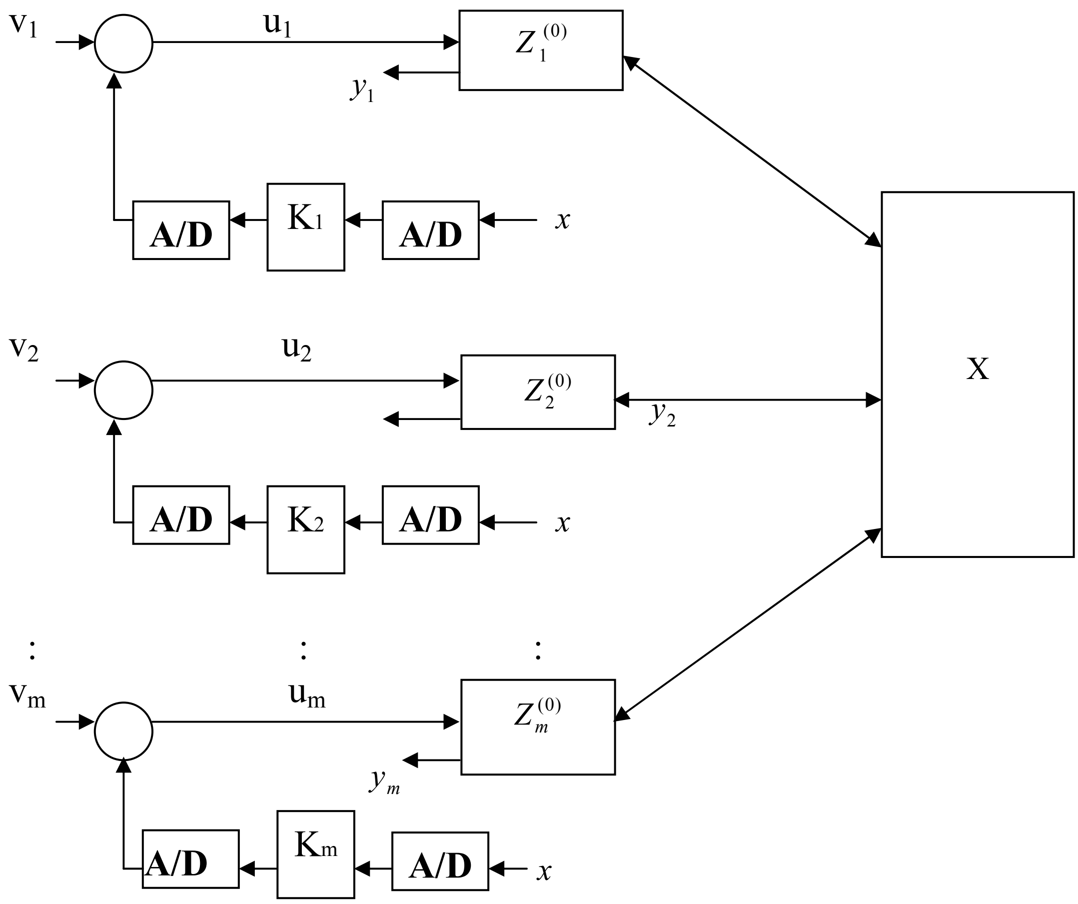
and $K_2 \sim K_m$ are existing feedback gains.

$G = \begin{bmatrix} \Gamma_1 & \Gamma_2 & \ldots. & \Gamma_m \end{bmatrix} = \Gamma$

$N(k) = \begin{bmatrix} v_1(k) \\ v_2(k) \\ : \\ v_m(k) \end{bmatrix}$, $v_1(k), v_2(k)\ldots$ are additional inputs.

and,
$$y_i(k) = C_{ri}x_r(k) + D_{ri}u_i(k) \tag{2b}$$

where $C_{ri} = -C_i A_{ii}^{-1} A_{i0}$; $D_{ri} = -C_i A_{ii}^{-1} B_i$; i = 1~m.



$*Z_1^{(0)} \sim Z_m^{(0)}$: Z_i of (1b) when $\varepsilon=0$

Figure 1. The decentralized singularly-perturbed actuator control system.

III. ROBUST CONTROL INVESTIGATION

In singularly-perturbed systems, the parameter ε reflects the changing of the state model by system uncertainties. Because of the values of ε is very small, it is assumed to be zero when the control design is proceeded. This is a system uncertainty of unmodel dynamics. How much effect caused on poles shifting by system uncertainties and how much toleration of systems are the issues we would like to look into.

Now, let look at a practical decentralized singularly-perturbed system model that is a fifth-order system with three first order subsystems and three inputs.

$$\begin{bmatrix} \dot{x} \\ \varepsilon\dot{z}_1 \\ \varepsilon\dot{z}_2 \\ \varepsilon\dot{z}_3 \end{bmatrix} = \begin{bmatrix} -0.5 & 0 & 0.1 & -0.2 & 0.1 \\ 0 & -1 & 0.1 & 0.3 & -0.2 \\ 0.4 & -0.3 & -0.5 & 0 & 0 \\ -0.4 & 0.4 & 0 & -0.45 & 0 \\ 0.35 & 0.3 & 0 & 0 & -0.4 \end{bmatrix} \begin{bmatrix} x \\ z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.4 & 0 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & 0.6 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad (3)$$

where x is a slow state vector that is second-order. $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

z_1, z_2, z_3 **are all fast state vectors and first-order individually.**

Therefore, when we set $\varepsilon = 0$, the system can be reduced to a second order system such as

$$\dot{x} = \begin{bmatrix} -0.1545 & -0.163 \\ -0.363 & -0.943 \end{bmatrix} x + \begin{bmatrix} 0.08 & -0.222 & 0.15 \\ 0.08 & 0.333 & -0.3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad (4)$$

Next, we digitize this reduced-order model to discrete-time domain with the sampling period 0.1.

$$x(k+1) = \begin{bmatrix} 0.985 & -0.0154 \\ -0.0344 & 0.9103 \end{bmatrix} x(k) + \begin{bmatrix} 0.0079 & -0.0223 & 0.0151 \\ 0.0075 & 0.0322 & -0.0289 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \\ u_3(k) \end{bmatrix} \quad (5)$$

Now, we will control the system from the subsystem one. By assuming the existing K2 and K3 are zeros, we can rewrite the model as

$$x(k+1) = \left\{ \begin{bmatrix} 0.985 & -0.0154 \\ -0.0344 & 0.9103 \end{bmatrix} + \begin{bmatrix} 0.0079 \\ 0.0075 \end{bmatrix} K_1 \right\} x(k) + GN(k) \quad (6)$$

where $G = \begin{bmatrix} 0.0079 & -0.0233 & 0.0151 \\ 0.0079 & 0.0322 & -0.0289 \end{bmatrix}$

$$N(k) = \begin{bmatrix} v_1(k) \\ v_2(k) \\ v_3(k) \end{bmatrix}$$

From the calculation, we know the locations of the poles in the reduced-order model are 0.9915 and 0.9038. If the desired locations of these poles are 0.9 and 0.8, after calculation, the necessary feedback gains $K_1 = [-30.6679 \quad 6.2717]$. Pole placement for the subsystem two and subsystem three can just follow the same procedure used in the subsystem one.

Now, we can perform robustness test on this system. Using the same technique and same conditions in the original full-order model with $\varepsilon = 0.001$**. The discrete-time full-order model would be like**

$$w(k+1) = \begin{bmatrix} 0.9842 & -0.0151 & 0.0002 & -0.0004 & 0.0003 \\ -0.0334 & 0.9102 & 0.0002 & 0.0004 & -0.0005 \\ 0.8072 & -0.559 & 0 & -0.0007 & 0.0005 \\ -0.9042 & 0.8239 & 0 & 0.0001 & -0.0006 \\ 0.837 & 0.6713 & 0.0003 & 0.0001 & -0.0001 \end{bmatrix} w(k) + \begin{bmatrix} 0.0077 & 0.00218 & 0.0147 \\ 0.0074 & -0.0315 & -0.0282 \\ 0.8017 & 0.0356 & 0.0281 \\ -0.0003 & -1.1574 & -0.0373 \\ 0.012 & -0.0045 & 1.4919 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \\ u_3(k) \end{bmatrix}$$

Where $w = \begin{bmatrix} x_1 \\ x_2 \\ z_1 \\ z_2 \\ z_3 \end{bmatrix}$

Now, we apply the feedback gains obtained from the reduced-order model for the desired pole locations at 0.9 and 0.8 to this full-order model. We will have the locations of the performing poles at 0.8999 and 0.8007. We can see the locations are very close to the desired locations. That shows robust control of the reduced-order controller.

Also, by assuming $y_1 = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ z_1 \\ z_2 \\ z_3 \end{bmatrix}$, we can have the system responses based on the subsystem one with h=0.1 and ε=0.001 as Figure 2. and Figure 3.:

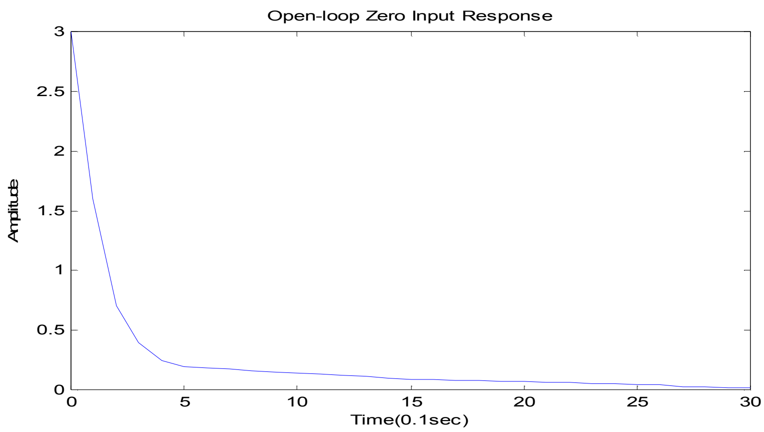


Figure 2. The open-loop zero-input response of the full-order system with the slow state poles at 0.9915 and 0.9038.

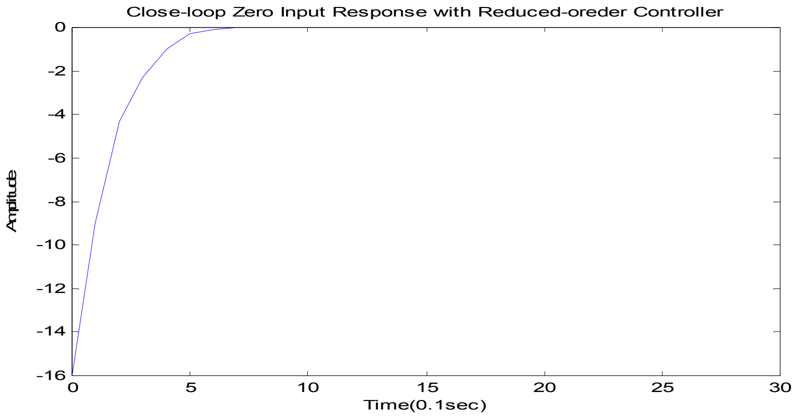


Figure 3. The close-loop zero-input response with the reduced-order controller shifting poles to 0.9 and 0.8.

We can also find out how the system tolerates system uncertainties by changing the parameter ε by a MATLAB program shown below:

```
% the robust control test of the above example.
% arbitrarily choose a small initial value for  ε.
z=0.0005;
% set the test for 50 loops.
for x=1:1:50;
% input known parameters of  the  Φ  and the  Γ.
a=0.4;b=-0.3;c=-0.5;
d=-0.4;e=0.4;f=-0.45;
g=0.35;h=0.3;i=-0.4;
j=0.4;k=-0.5;l=0.6;.
% express how these parameters affected by  ε.
a=a./z;b=b./z;c=c./z;
d=d./z;e=e./z;f=f./z;
g=g./z;h=h./z;i=i./z;
j=j./z;k=k./z;l=l./z;
% the  Φ  of the full-order state model.
A=[-0.5 0 0.1 -0.2 0.1;
      0 -1 0.1 0.3 -0.2;
      a   b   c   0   0;
      d   e   0   f   0;
      g   h   0   0   i];
% the  Γ  of the full-order state model.
B=[0 0 0;0 0 0;j 0 0;0 k 0;0 0 l];
% perform analog to digital transformation with sampling rate 0.1.
[G,P]=c2d(A,B,0.1);
```

```
% the existing controlling feedback gain of the reduced-order controller.
K=[-30.6679 6.2717 0 0 0];
% the Γ1 of the discrete-time state model.
P1=P(:,1);
% the new Φ of the discrete-time state model
M=G+P1*K;
% the pole locations of  the closed-loop system controlled by the subsystem one .
Y=eig(M);
% the dominant pole locations of using the reduced-order controller.
Y(1:2,1)
% move to next test with the interval 0.0055.
z=z+0.0055
end
```

Table 1 shows how the poles shift when the value of ε changes.

Table 1. The robust control test.

ε	**Poles**
5.0000e-004	0.8999, 0.8003
0.0060	0.9001, 0.7954
0.0115	0.9003, 0.7902
0.0170	0.9005, 0.7845
0.0225	0.9007, 0.7781
0.0280	0.9009, 0.7707
0.0335	0.9011, 0.7619
0.0390	0.9013, 0.7511
0.0445	0.9015, 0.7368

Every system has difference tolerance from system uncertainties. In this case, we assume the system performance allows 0.02 shift at each pole location. Then, when $\varepsilon < 0.017$, we can have a robust control system. The reduced-order controllers that perform inside this bound are called robust, decentralized reduced-order controllers.

The same procedure can be used in the subsystem two and the subsystem three for the robust control test.

IV. RELIABLE CONTROL INVESTIGATION

In decentralized systems, reliable control is obtained, if every controller of every subsystem is able to stabilize the system. Even if one of the controllers fails, the rest of the controllers can still control the overall system. Therefore, the system can avoid breakdown problem. Siljack presents the approach of designing a separate stabilizing controller for each control channel. In such a structure, the system is called a multiple control system. It has been established that the system possessing this type of structure has build-in reliability properties [4]. Figure 4 shows the structure of a multiple control scheme with two sub-systems.

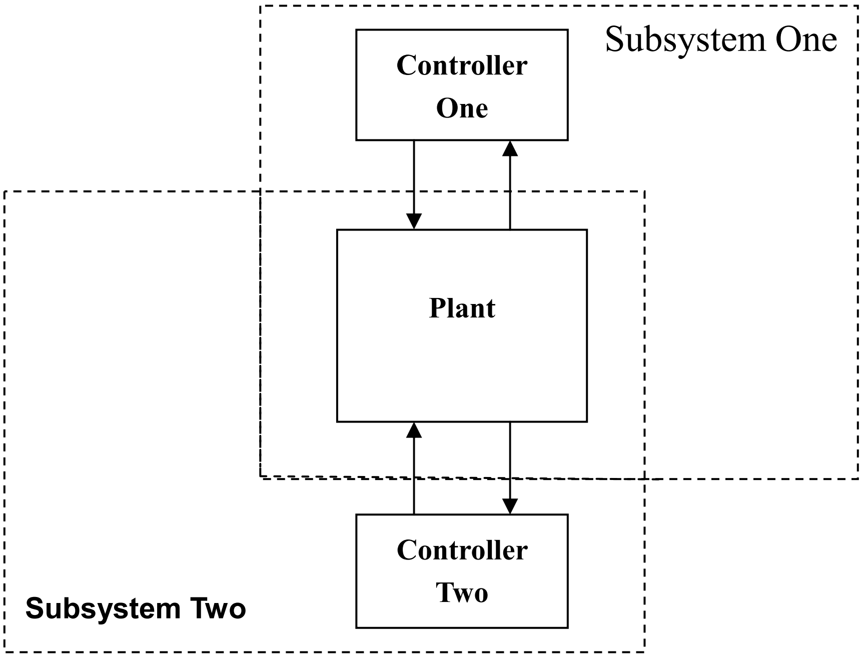


Figure 4. A multiple control scheme.

On the contrary, the other type of decentralized control scheme that has a state equation (7), [9]-[11]. In this type of decentralized systems, the controller of each channel does not have ability to control overall state variables. In (7), the term $u_i(k)$ influences $\sum_{\substack{j=1 \\ j\neq i}}^{N} \Phi_{ij} x_j(k)$, but it is unable to control the poles of $x_j(k)$. $u_i(k)$ can only control $x_i(k)$.

$$x_i(k+1) = \Phi_i x_i(k) + \Gamma_i u_i(k) + \sum_{\substack{j=1 \\ j\neq i}}^{N} \Phi_{ij} x_j(k) \tag{7}$$

where i is the number of the controlled unites.

j is the number of the subsystems.

and, $i = j$.

From the concepts of multiple control systems, to design a reliable decentralized control system can be based on the structure of decentralized control systems.

Now, let us look at the structure of a decentralized singularly-perturbed system shown in equation (8). Equation (8) is the state equation of the subsystem one in decentralized, full-order singularly-perturbed systems. It can be found by the same method as equation (2). The reason why equation (8) is discussed for reliable control is to see whether the controller K_1 can control overall state variables.

$$w(k+1) = [\Phi_N + \Gamma_1 K_1(k)]w(k) + GN(k) \tag{8}$$

where $\Phi_N = (\Phi + \Gamma_2 K_2 + \Gamma_3 K_3 + \ldots \Gamma_m K_m)$

,and $K_2 \sim K_m$ are existing feedback gains.

$G = \begin{bmatrix} \Gamma_1 & \Gamma_2 & \ldots & \Gamma_m \end{bmatrix} = \Gamma$

$N(k) = \begin{bmatrix} v_1(k) \\ v_2(k) \\ \vdots \\ v_m(k) \end{bmatrix}$; $v_1(k), v_2(k) \ldots$ are additional inputs

After choosing an appropriate value for K_1, the matrix $\Gamma_1 K_1(k)$ will be absorbed to the matrix Φ_N; therefore, a $n \times n$ matrix $[\Phi_N + \Gamma_1 K_1(k)]$ that contains the existing poles of the system is obtained. In other words, the controller K_1 can control the overall state variables.

From (8), the input of the subsystem one

$$u_1(k) = K_1 w(k) + v_1(k) \tag{9}$$

It obviously shows that the input $u_1(k)$ can control overall state variables. It is likewise in the rest of the inputs. In addition, the stabilizing controller has been developed by the Riccati equation approach [1]. Therefore, these two concepts conclude that the controller of each channel can stabilize the overall state variables. This is same as the idea of a multiple control system proposed by Siljack, and the concept also fulfill to evaluate the reliable and robust systems [12].

V. CONCLUSION

The control scheme described in this research is under investigation for robustness and reliability. A procedure is developed to test if a decentralized singularly-perturbed actuator system is a robust control system. The bound of robust control will be found. It helps us to understand how much the system can tolerate system uncertainties. Consequently, appropriate protections can be applied to prevent malfunctions of the systems.

The control scheme described in this research uses the structural reliability concept to construct a reliable control system. By comparing with other reliable control designs that require additional devices, this design provides the most economical scheme to have a reliable control system. Moreover, the control scheme is not complicated by the goal of reliable control, too. The only condition of this reliable control design is that the controller of each channel has to be a robust stabilizing controller. That already has been achieved in [1] .

REFERENCES

[1] Kai-chao Yao "Digital Observer-based Controller Design of Decentralized Actuator Systems", First International NAISO Congress on Autonomous Intelligent Systems (ICAIS '2002'), Australia, CD #100028-02-KY-042, Feb., 2002.

[2] R. J. Veillette, J. V. Medanic and W. R. Perkinsa. "Design of Reliable Control Systems," IEEE Tran. Automat. Contr., Vol. 37, pp. 290~304, 1992.

[3] Richard C. D. and Robert H. B., Modern Control Systems, Addison Wesley Longman Inc, CA, 1998.

[4] D. D. Siljak, "Reliable Control Using Multiple Control System," Int. J. Contr., Vol. 31, No. 2, pp. 303~329,1980.

[5] G. C. Goodwin, Maria M. Seron and Mario E. Salgado, "H_2 Design of Decentralized Controllers," American Control Conference, University of Newcastle: ECE Depart., pp. 3826~3830, 1999.

[6] R. A. Date and J. H. Chow, "A Reliable Coordinated Decentralized Control System Design," In Proc. 28th Conf. Decision Contr., Tampa, FL, pp. 1295~1300, 1989.

[7] M. Mariton and P. Bertrand, "Improved Mutiplex Control Systems: Dynamic Reliability and Stochastic Optimality," Int. J.Contr., Vol. 44, pp. 219~234, 1986.

[8] M. Vidyasagar and N. Viswanadham, "Reliable Stabilization Using A Multicontroller Configuration," Automatica, Vol. 21, pp. 599~602, 1985.

[9] D. D. Siljak and M. B. Vukcevic, "Decentralization, Stabilization, and Estimation of Large-scale Linear System," IEEE Trans. Automat. Contr., pp 363~366, 1976.

[10] K. Yasuda, "Decentralized Optimal Control for Large-scale Interconnected Systems," Control and Dynamic System, Academic Press, NY, pp. 139~162, 1986.

[11] Cheng-Fa Cheng, "Decentralized Robust Controller and Observer Design for Uncertain Large-scale Systems," Thesis of P.H.D., EE Depart., National Chiao-Tung University,1993.

[12] Pierce, S. G., Ben-Haim, Y., Worden, K., Manson, G.," Evaluation of Neural Network Robust Reliability Using Information-Gap Theory", Neural Networks, IEEE Transactions on, pp 1349 – 1361, Volume 17, Issue 6, Nov. 2006.