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# 使用線性矩陣不等式法探討強健時延不確定性系統飽合致動器的強健穩定化

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## 摘　要

本論文探討一組不確定時變時延系統飽合致動器的穩定化準則。根據李亞普若夫汎函與結合線性矩陣不等式技巧，簡單且改善時延相關強健穩定準則被提出。一個數值範例說明這方法的有效性與可應用性。

**關鍵字：時延系統、時延相關性、致動器邊限、線性矩陣不等式。**

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# Robust Stabilization for Uncertain Time-delay Systems with Saturating Actuators-LMI Approach

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## Abstract

This paper provides new stabilization criteria for a class of uncertain linear time-delay systems with saturating actuators for time-varying delays. Based on Lyapunov-Krasovskii functionals combined with LMI techniques, simple and improved delay-dependent robust stability criteria, which are given in terms of quadratic forms of state and LMI, are derived. A numerical example is given to illustrate the effectiveness and applicability.

***Index Terms***- time-delay systems, delay dependence, actuator constraint, linear matrix inequality.

## I. Introduction

Time delays and/or saturating actuators are frequently encountered in various engineering systems, and their existence is often the source of instability. Therefore, it is very desirable for the control system design to investigate the problem of the stabilization of systems with time delay and/or saturating actuators. The stabilization problem of time delay systems has been of interest to researchers over the past decade [1]-[2]. In control system design, the limited power supply is in the form of a saturating actuator in a practical system, hence the actuator is nonlinear. Several authors have discussed the problem of stabilization of the system with saturating actuator [3]-[5].

A major subject in the analysis of linear dynamical systems with time-delay is related to the stability. The criteria for asymptotic stability of such systems can be classified as the one of delay independent, which do not include any information on the size of delay, for example, [6]-[7]. Tarbouriech and Garcia [7] presented to study the problem of stabilization of neutral systems with saturating state-feedback control law. The approach adopted is based on the use of some Lyapunov-Krasovskii functionals with delay-independent stabilization of linear time-invariant time-delay systems. The delay dependent includes such information on the size of delay, for example, [8]-[9]. Han and Ni [9] addressed uncertain saturated systems with pointwise and distributed time-varying delays. More precisely, using the Lyapunov-Razumikhin function approach proposes the upper bounds on the time varying delays such that the uncertain system is robustly asymptotically stabilizable. The stability criteria have been proposed via LMI approach [2], [10]-[12]. Niculescu [13] proposed a new $H_\infty$ memoryless control and $\alpha$ stability constraint for time delay systems via Lyapunov-Krasovskii functional and LMI approach.

This paper provides new stabilization criteria for a class of uncertain linear time-delay systems with saturating actuators for time-varying delays. The system parameter uncertainties are unknown but bounded, and the delays are time-varying. Based on Lyapunov–Krasovskii functionals combined with LMI techniques, we obtain simple and improved delay-dependent stabilization criteria. Our results, which are given in terms of quadratic forms of state and LMI, are more informative. The proposed schemes in the paper are applicable to robust control design.

## II. System Description

Consider the following uncertain time-delay systems with saturating actuator described by

$$\begin{aligned}\dot{x}(t) &= (A_0 + \Delta A_0(x,t))x(t) \\ &\quad + \sum_{i=1}^{k}(A_i + \Delta A_i(x,t))x(t-h_i(t)) \\ &\quad + (B + \Delta B(x,t))Sat(u(t)) \qquad (1)\end{aligned}$$

$$x(t) = \phi(t), \quad \forall t \in [-h, 0] , \tag{2}$$

where $x(t) \in R^n$ is the state vector and $A_j, j = 0, 1, \Lambda, k$, are known constant matrices with appropriate dimensions, $\Delta A_j(x,t)$ and $\Delta B(x,t)$, $j = 0, 1, \Lambda, k$, are matrix functions representing the uncertainties in the matrices $A_j$, $j = 0, 1, \Lambda, k$, and $B$.

$$\Delta A_j(x,t) = D_j F_j(x,t) E_j, \; j = 0, 1, \Lambda, k, \tag{3}$$

$$\Delta B(x,t) = D_0 F_0(x,t) E_b , \tag{4}$$

where $F_j(x,t) \in R^{k_j \times g_j}$ are unknown real time-varying matrices with Lebesgue measure elements bounded by

$$F_j^T(x,t) F_j(x,t) \le I, \forall t, j = 0, 1, \Lambda, k, \tag{5}$$

and $D_j$, $E_j$ and $E_b$ are known real constant matrices, $h_i(t), i = 1, \Lambda, k,$ are the unknown time-varying delay terms, but bounded $0 \le h_i(t) \le h$, $\dot{h}_i(t) \le d_i < 1$, $\phi(t)$ is a smooth vector-valued initial function in $-h \le t \le 0$. The operation range of nonlinear saturation $Sat(u_i(t))$ is considered inside the sector $[\alpha_i, 1]$, which means that the graph of nonlinearity lies between two straight lines passing through the origin with slopes $\alpha_i$ and 1, respectively, where $0 \le \alpha_i \le 1$, $, i = 1, \Lambda, m$. Let $u_{ih}$ and $u_{il}$ be the saturating values, such that

$$Sat(u_i(t)) = \begin{cases} u_{ih} & if \quad 0 < u_i(t) \le u_{ih} \\ u_i(t) & if \quad u_{il} \le u_i(t) \le u_{ih} \\ u_{il} & if \quad u_i(t) \le u_{il} < 0 \end{cases} , \tag{6}$$

where the values of $u_{ih}$ and $u_{il}$ are chosen corresponding to the limitations with the following properties.

$$\begin{aligned} & Sat(u_i(t)) - \frac{1}{2}(1 + a_i) u_i(t) \\ & = \Delta t'_i u_i(t), i = 1, 2, \Lambda\, m, \end{aligned} \tag{7}$$

this implies that

$$Sat(u(t)) - Wu(t) = \Delta T' u(t) , \tag{8}$$

where

$$Sat(u(t)) = [Sat(u_1(t)),\Lambda\ , Sat(u_m(t))]^{\mathrm{T}},$$

$$W = diag[\frac{1}{2}(1+a_1),\Lambda\ ,\frac{1}{2}(1+a_m)],$$

$$\Delta T' = diag[\Delta t'_1,\Lambda\ ,\Delta t'_m].$$

Where $\Delta t'_i$ is a real number which varying between $-\frac{1}{2}(1-a'_i)$ and $\frac{1}{2}(1-a'_i)$, where

$$a_i < a'_i < 1. \tag{9}$$

We assume $(A, B)$ is controllable. In this paper, we pay attention to the following state-feedback controller

$$u(t) = -Kx(t), \tag{10}$$

where the state feedback matrix $K$ has the appropriate dimension.

From (1), (8) and (10), we can obtain the resulting of uncertain time-delay systems with saturating actuator

$$\begin{aligned}
\dot{x}(t) &= (A_0 + \Delta A_0(x,t))x(t) \\
&\quad + \sum_{i=1}^{k}(A_i + \Delta A_i(x,t))x(t-h_i(t)) \\
&\quad + (B + \Delta B(x,t))[Sat(u(t)) - Wu(t)] \\
&\quad + (B + \Delta B(x,t))(Wu(t)) \\
&= (A_F + \Delta A_F(x,t))x(t) \\
&\quad + \sum_{i=1}^{k}(A_i + \Delta A_i(x,t))x(t-h_i(t))
\end{aligned} \tag{11}$$

where

$$A_F = A_0 - B(W + \Delta T')K,$$
$$\Delta A_F(x,t) = \Delta A_0(x,t) - \Delta B(x,t)(W + \Delta T')K.$$

The main aim of this paper is to develop delay-dependent criteria for robust stability of the uncertain time delay system (1). More specifically, our objective is to determine bounds for the time delay by using different Lyapunov-Krasovskii functionals and LMI methods.

The following matrix inequality will be essential for the proofs.

**Lemma 1 [14]**:

Let $D, E$ and $F$ be real matrices of appropriate dimensions with $\|F\| \le 1$, then we have the following:

$$DFE + E^T F^T D^T \le \varepsilon^{-1} D D^T + \varepsilon E^T E, \tag{12}$$

for any scalar $\varepsilon > 0$.

## Ⅲ. Main Results

In this section, we describe our method for determining the robust stabilization criteria of uncertian time-delay system (1)-(3) and (11). The main results are given in the following theorems.

Theorem 1: Consider the uncertain delay system (11) with $\Delta A_j(x,t) = D_j F_j(x,t) E_j,\ j = 0,1,\Lambda\ ,k, \Delta B(x,t) = D_0 F_0(x,t) E_b, \|F_j\| \le 1$, for all delays $h_i(t) \in [0,h]$. For given $\alpha > 0$, this system is robustly stable if there exist symmetric and positive-definite matrices $P > 0,\ R_i > 0, i = 1,\Lambda\ ,k$, and scalar $\varepsilon_j > 0,\ j = 0,1,\Lambda\ ,k$, such that the following LMI holds:

$$\begin{bmatrix} S & M_0 & M_1 & M_2 & M_3 \\ M_0^T & -N_0 & 0 & 0 & 0 \\ M_1^T & 0 & N_1 & 0 & 0 \\ M_2^T & 0 & 0 & -N_2 & 0 \\ M_3^T & 0 & 0 & 0 & -N_3 \end{bmatrix} < 0, \tag{13}$$

where

$$\begin{aligned} S = {} & A_F X^T + A_F^T X + 2\alpha X \\ & + X^T \left(\sum_{i=1}^{k} R_i + \varepsilon_0 E_0^T E_0\right) X \\ & - \varepsilon_1 Y^T [E_b(\Delta T' + W)]^T [E_b(\Delta T' + W)] Y, \end{aligned}$$

$X = P^{-1},\ Y = KX$,

$M_0 = D_0,\ N_0 = \varepsilon_0,\ M_1 = D_0,\ N_1 = \varepsilon_1,\ M_2 = [D_1, \Lambda\ , D_k]$,

$$\begin{aligned} & N_2 = diag\left[e^{-2\alpha h}\varepsilon_2, \Lambda\ , e^{-2\alpha h}\varepsilon_{k+1}\right],\ M_3 = \left[e^{\alpha h} A_1, \Lambda\ , e^{\alpha h} A_k\right], \\ & N_3 = diag\left[(1-d_1)R_1 - \varepsilon_2 E_1^T E_1\right. \\ & \qquad \left., \Lambda\ ,\ (1-d_k) R_k - \varepsilon_{k+1} E_k^T E_k\right]. \end{aligned}$$

Proof: Via the state transformation matrix

$$z(t) = e^{\alpha t} x(t),\ t > 0 \tag{14}$$

where $\alpha > 0$ is stability degree, transform (1) into

$$
\begin{aligned}
\dot{z}(t) &= \alpha e^{\alpha t} x(t) + e^{\alpha t} \dot{x}(t) \\
&= (A_F + \Delta A_F + \alpha I) z(t) \\
&\quad + \sum_{i=1}^{k} e^{\alpha h_i(t)} (A_i + \Delta A_i) z(t - h_i(t)).
\end{aligned} \tag{15}
$$

First, let us consider the time-delay system of (1), using the Lyapunov-Krasovskii functional candidate in the following form [15]-[17], then we can write

$$
\begin{aligned}
& V(z(t), z(t-h_i(t))) \\
&= z^T(t) P z(t) \\
&\quad + \sum_{i=1}^{k} \int_{t-h_i(t)}^{t} z^T(\theta) R_i z(\theta) d\theta.
\end{aligned} \tag{16}
$$

The time derivative of (16) along the trajectory of (1) is given by

$$
\begin{aligned}
\dot{V} &= \dot{z}^T(t) P z(t) + z^T(t) P \dot{z}(t) + \sum_{i=1}^{k} z^T(t) R_i z(t) \\
&\quad - \sum_{i=1}^{k} (1 - \dot{h}_i(t)) z^T(t - h_i(t)) R_i z(t - h_i(t)).
\end{aligned} \tag{17}
$$

Using Lemma 1, we have

$$
\dot{V} \le U^T \Omega U < 0, \tag{18}
$$

where

$$
U = [z^T(t), z^T(t-h_1(t)), \Lambda, z^T(t-h_k(t))]^T, \tag{19}
$$

$$
\Omega = \begin{bmatrix} M & \mathrm{G}^{\mathrm{T}} \\ G & \mathrm{L} \end{bmatrix}. \tag{20}
$$

where

$$
\begin{aligned}
M &= (A_F + \alpha I)^T P + P(A_F + \alpha I) \\
&\quad + \sum_{i=1}^{k} R_i + (\varepsilon_0 - \varepsilon_1) P D_0 D_0^T P + \varepsilon_0^{-1} E_0^T E_0 \\
&\quad - \varepsilon_1^{-1} [E_b(\Delta T' + W)K]^T [E_b(\Delta T' + W)K] \\
&\quad + \sum_{i=1}^{k} \varepsilon_{i+1}^{-1} e^{2\alpha h_i} P D_i D_i^T P,
\end{aligned} \tag{21}
$$

$$
G^T = \left[ e^{\alpha h_1} P A_1, \Lambda, e^{\alpha h_k} P A_k \right], \tag{22}
$$

$$L = -diag\big[(1-d_1)R_1 - \varepsilon_1 E_1^T E_1$$
$$,\Lambda\ ,\ (1-d_k)R_k - \varepsilon_k E_k^T E_k\big]. \qquad (23)$$

Finally, letting $X = P^{-1}$ and $Y = KX$ and using the Schur complements, with some efforts, we can show that (13) guarantees the negativeness of $V^{\&}$ whenever $U$ in (19) is not zero, which immediately implies the asymptotic stabilization of the system (1).

**Remark 1**:

If we let $k = 1$, then system (1) is a single state-delay system, and the (13) can be transformed into (24) as LMI problem on a single state delay

$$\begin{bmatrix} \overline{S} & \overline{M}_0 & \overline{M}_1 & \overline{M}_2 & \overline{M}_3 \\ \overline{M}_0^T & -\overline{N}_0 & 0 & 0 & 0 \\ \overline{M}_1^T & 0 & \overline{N}_1 & 0 & 0 \\ \overline{M}_2^T & 0 & 0 & -\overline{N}_2 & 0 \\ \overline{M}_3^T & 0 & 0 & 0 & -\overline{N}_3 \end{bmatrix} < 0, \qquad (24)$$

where

$$\overline{S} = A_F X^T + A_F^T X + 2\alpha X$$
$$+ X^T (R_1 + \varepsilon_0 E_0^T E_0) X$$
$$- \varepsilon_1 Y^T [E_b(\Delta T' + W)]^T [E_b(\Delta T' + W)] Y$$

$X = P^{-1}$, $Y = KX$,

$\overline{M}_0 = D_0$, $\overline{N}_0 = \varepsilon_0$, $\overline{M}_1 = D_0$, $\overline{N}_1 = \varepsilon_1$ , $\overline{M}_2 = D_1$,

$\overline{N}_2 = e^{-2\alpha h}\varepsilon_2$, $\overline{M}_3 = e^{\alpha h} A_1$,

$\overline{N}_3 = (1-d_1)R_1 - \varepsilon_2 E_1^T \mathrm{E}_1$.

**Remark 2**:

In Theorem 2, we will consider stabilization criteria via a different choice of Lyapunov-Krasovskii functional combined with LMI techniques.

**Theorem 2**: Considering the uncertain delay system (11), for all delays $h_i(t) \in [0, h]$. For given $\alpha > 0$, this system is robustly stable if there exist symmetric and positive-definifte matrices $P > 0, R_i > 0, Q_{i0} > 0,\ i = 1,\Lambda\ ,k$ and scalar $\varepsilon_j > 0,\ j = 0,\Lambda\ ,k$, such that the following LMI holds:

$$
\begin{bmatrix}
\hat{S} & \hat{M}_0 & \hat{M}_1 & \hat{M}_2 & \hat{M}_3 \\
\hat{M}_0^T & -\hat{N}_0 & 0 & 0 & 0 \\
\hat{M}_1^T & 0 & \hat{N}_1 & 0 & 0 \\
\hat{M}_2^T & 0 & 0 & -\hat{N}_2 & 0 \\
\hat{M}_3^T & 0 & 0 & 0 & -\hat{N}_3
\end{bmatrix} < 0, \qquad (25)
$$

where

$$
\begin{aligned}
&\hat{S} \\
&= A_F X^T + A_F^T X + 2\alpha X \\
&\quad + X^T (\sum_{i=1}^{k} R_i + \varepsilon_0 E_0^T E_0) X \\
&\quad - \varepsilon_1 Y^T [E_b(\Delta T' + W)]^T [E_b(\Delta T' + W)] Y \\
&\quad + \sum_{i=1}^{k} h_i Q_{io} \ ,
\end{aligned}
$$

$X = P^{-1}$, $Y = KX$ ,

$\hat{M}_0 = D_0$, $\hat{N}_0 = \varepsilon_0$, $\hat{M}_1 = D_0$, $\hat{N}_1 = \varepsilon_1$, $\hat{M}_2 = [D_1, \Lambda\ , D_k]$,

$\hat{N}_2 = diag[e^{-2\alpha h}\varepsilon_2, \Lambda\ , e^{-2\alpha h}\varepsilon_{k+1}]$, $\hat{M}_3 = [e^{\alpha h} A_1, \Lambda\ , e^{\alpha h} A_k]$,

$$
\begin{aligned}
\hat{N}_3 = diag\big[&(1-d_1)R_1 - \varepsilon_2 E_1^T \mathrm{E}_1 \\
&, \Lambda\ , \ (1-d_k) R_k - \varepsilon_{k+1} E_k^T E_k\big]
\end{aligned}
$$

**Proof**: Considering the time-delay system of (1), using the improved Lyapunov-Krasovskii functional candidate in the following form, we can write

$$
\begin{aligned}
&V(z(t), z(t-h_i(t))) \\
&= z^T(t) P z(t) + \sum_{i=1}^{k} \int_{t-h_i(t)}^{t} z^T(\theta) R_i z(\theta) d\theta \\
&\quad + \sum_{i=1}^{k} \int_{-h_i(t)}^{0} \int_{t+\beta}^{t} z^T(\rho) Q_{i0} z(\rho) d\rho\, d\beta . \qquad (26)
\end{aligned}
$$

$$\begin{aligned}\dot{V} =\ & \dot{z}^T(t)Pz(t) + z^T(t)P\dot{z}(t) + \sum_{i=1}^{k} z^T(t)R_i z(t) \\ & - \sum_{i=1}^{k}(1-\dot{h}_i(t))z^T(t-h_i(t))R_i z(t-h_i(t)) + \\ & \sum_{i=1}^{k} z^T(t)h_i(t)Q_{i0}z(t) - \sum_{i=1}^{k}\int_{t-h_i(t)}^{t} z^T(\beta)Q_{i0}z(\beta)d\beta + \\ & \sum_{i=1}^{k}\dot{h}_i(t)\int_{t-h_i(t)}^{t} z^T(\rho)Q_{io}z(\rho)d\rho. \end{aligned} \tag{27}$$

Using Lemma 1, we obtain

$$\begin{aligned}\dot{V} \le\ & X^T \bar{T} X - \sum_{i=1}^{k}\int_{t-h_i(t)}^{t} z^T(\beta)Q_{i0}z(\beta)d\beta \\ & + \sum_{i=1}^{k}\dot{h}_i(t)\int_{t-h_i(t)}^{t} z^T(\rho)Q_{io}z(\rho). \end{aligned} \tag{28}$$

Since the last two terms $-\sum_{i=1}^{k}\int_{t-h_i(t)}^{t} z^T(\beta)Q_{i0}z(\beta)d\beta + \sum_{i=1}^{k}\dot{h}_i(t)\int_{t-h_i(t)}^{t} z^T(\rho)Q_{io}z(\rho)$ in (28) is negative-definite [18], we have

$$\dot{V} \le U^T \bar{\Omega} U < 0, \tag{29}$$

where

$$U = [z^T(t), z^T(t-h_1(t)), \ldots, z^T(t-h_k(t))]^T, \tag{30}$$

$$\bar{\Omega} = \begin{bmatrix} \bar{M} & \mathrm{G}^{\mathrm{T}} \\ G & \bar{\mathrm{L}} \end{bmatrix}, \tag{31}$$

$$\begin{aligned} & \bar{M} \\ & = (A_F + \alpha I)^T P + P(A_F + \alpha I) \\ & \quad + \sum_{i=1}^{k} R_i + (\varepsilon_0^{-1} - \varepsilon_1^{-1})P D_0 D_0^T P + \varepsilon_0 E_0^T E_0 \\ & \quad - \varepsilon_1 [E_b(\Delta T' + W)K]^T [E_b(\Delta T' + W)K] \\ & \quad + \sum_{i=1}^{k} \varepsilon_{i+1}^{-1} e^{2\alpha h_i} P D_i D_i^T P + \sum_{i=1}^{k} h_i Q_{i0}, \end{aligned} \tag{32}$$

$$G^T = \left[e^{\alpha h_1} P A_1, \ldots, e^{\alpha h_k} P A_k\right], \tag{33}$$

$$\begin{aligned}\bar{L} = -diag\big[ & (1-d_1)R_1 - \varepsilon_2 E_1^T E_1 \\ & , \ldots,\ (1-d_k)R_k - \varepsilon_{k+1} E_k^T E_k \big]. \end{aligned} \tag{34}$$

Finally, letting $X = P^{-1}$ and $Y = KX$, using the Schur complements, with some efforts, we can show that (25) guarantees the negativeness of $\dot{V}$ whenever $U$ in (30) is not zero, which immediately implies the asymptotic stabilization of the system (1).

**Remark 3**:

If we let $k = 1$, then system (1) is a single state-delay system, and (25) can be transformed into (35) as LMI problem on a single state delay,

$$\begin{bmatrix} \tilde{S} & \tilde{M}_0 & \tilde{M}_1 & \tilde{M}_2 & \tilde{M}_3 \\ \tilde{M}_0^T & -\tilde{N}_0 & 0 & 0 & 0 \\ \tilde{M}_1^T & 0 & \tilde{N}_1 & 0 & 0 \\ \tilde{M}_2^T & 0 & 0 & -\tilde{N}_2 & 0 \\ \tilde{M}_3^T & 0 & 0 & 0 & -\tilde{\mathrm{N}}_3 \end{bmatrix} < 0, \tag{35}$$

where

$$\begin{aligned} &\tilde{S} \\ &= A_F X^T + A_F^T X + 2\alpha X \\ &\quad + X^T (R_1 + \varepsilon_0 E_0^T E_0) X \\ &\quad - \varepsilon_1 Y^T [E_b(\Delta T' + W)]^T [E_b(\Delta T' + W)] Y \\ &\quad + hQ, \end{aligned}$$

$$X = P^{-1}, \ \ Y = KX,$$

$$\tilde{M}_0 = D_0, \ \tilde{N}_0 = \varepsilon_0, \ \tilde{M}_1 = D_0, \ \tilde{N}_1 = \varepsilon_1, \ \tilde{M}_2 = D_1,$$

$$\tilde{N}_2 = e^{-2\alpha h} \varepsilon_2, \ \tilde{M}_3 = e^{\alpha h} A_1,$$

$$\tilde{N}_3 = (1 - d_1) R_1 - \varepsilon_2 E_1^T \mathrm{E}_1.$$

## IV. Example

Consider the following uncertain time delay system:

$$\begin{aligned} \dot{x}(t) &= [A_0 + \Delta A_0(t)] x(t) + [A_1 + \Delta A_1(t)] x(t - h(t)) \\ &\quad + B Sat(u(t)), \end{aligned} \tag{36}$$

where

$$A_0 = \begin{bmatrix} -2 & 0 \\ 1 & -3 \end{bmatrix},\ A_1 = \begin{bmatrix} -1 & 0 \\ -0.8 & -1 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

and $\Delta A_0(t)$ and $\Delta A_1(t)$ are uncertain matrices satisfying

$$\|\Delta A_0(t)\| \le 0.2,\ \|\Delta A_1(t)\| \le 0.2,\ \forall t$$

$$D_0 = D_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},\ E_0 = E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The nonlinear saturating characteristic belongs to the sector $0.3 \le a_i < 1$.

Applying Theorem 1 to this uncertain time-delay system, if we choose $a'_i = 0.5$, it is found, using the software package LMI Lab, that this system is robustly stabilizable for the time-varying time delay $h(t)$, $0 \le h(t) \le 4.0813$, with a feedback control $K = [5.1926 \quad 4.7212]$. However, from Theorem 2, we get $h(t)$, $0 \le h(t) \le 4.4206$ and a control $K = [0.2209 \quad 1.3031]$. Thus, Theorem 2 improves stabilization criteria mentioned in Theorem 1. The simulations of the above closed loop system are depicted in Figure 1 and Figure 2, respectively.

## V. Conclusion

This paper deals with the problem of robust stabilization criteria for a class of uncertain linear time-delay saturating actuator systems. Based on Lyapunov-Krasovskii functionals combined with LMI techniques, simple and improved delay-dependent robust stabilization criteria are derived. The robust stabilization criteria do not involve any supplementary constraints on the system matrices and are analytical from the system parameters. A numerical example shows that the presented method is feasible and effective to robust control design.

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Figure 1. The simulation of Theorem 1 for $h(t) = 4.08$

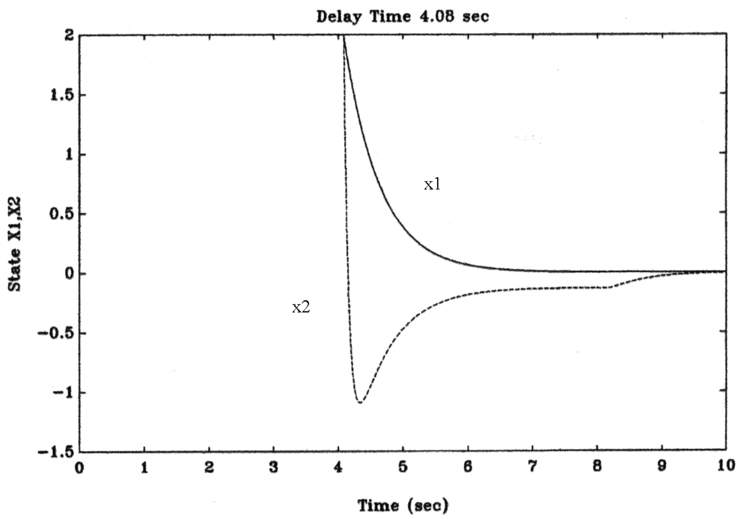
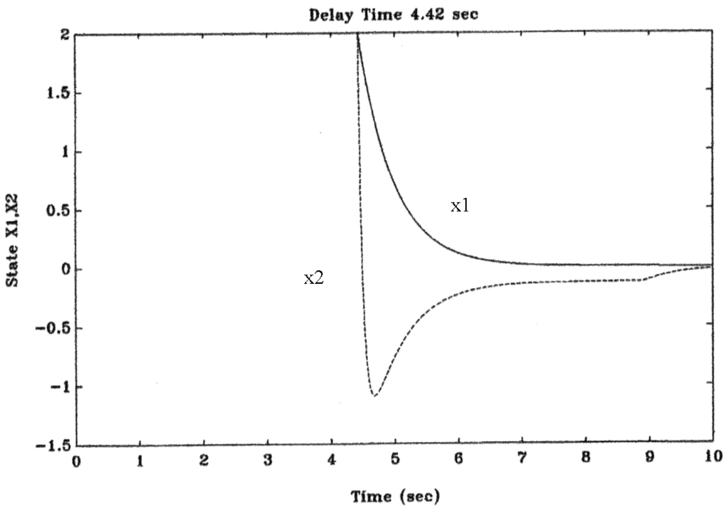



Figure 2. The simulation of Theorem 2 for $h(t) = 4.42$



Delay Time 4.42 sec
State X1,X2
Time (sec)
x1
x2