工業教育與技術學刊
第三十四期　民國九十七年　第 135～148 頁
Journal of Industrial Education and Technology
Vol. 34. December. 2008, PP. 135～148

不確定性非線性隨機中性時延系統強健 H_∞ 控制

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摘　要

本論文處理不確定性隨機中性系統的強健穩定性及 H_∞ 控制問題。非線性項假設為整體 Lipschitz 條件及含括在擾動項中。首先探討狀態迴授控制器設計，導致不確定性中性系統達到時延相關的穩定度化。接著處理不確定性隨機中性系統於擾動衰減下會保證 H_∞ 的有界邊限。所得結果以線性矩陣不等式來表示。最後，兩個範例說明所提理論的有效性與正確性。

關鍵字：不確定性、時延系統、線性矩陣不等式、隨機穩定度

Journal of Industrial Education and Technology
Vol. 34. December. 2008, PP. 135～148

Robust H_∞ Control for Uncertain Nonlinear Stochastic Neutral Systems with State Delays

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Abstract

This paper deals with the problem of robust stability and robust H_∞ control for a class of uncertain neutral systems. The nonlinearities are assumed to satisfy the global Lipschitz conditions and appear in the term of perturbation. Attention first is focused on investigating a sufficient condition for designing a state feedback controller which stabilizes the uncertain neutral system under consideration. Robust stabilization is dependent on delay. Then, we show that guarantees an H_∞-norm bound constraint on the disturbance attenuation. The proposed results are given in terms of linear matrix inequalities. Two examples are worked out to illustrate the validness of the theoretical results.

Key Words: uncertainty, time-delay systems, linear matrix inequalities, stochastic stability

Ⅰ. Introduction

Time-delay frequently appears in many control systems (such as aircraft, chemical or process control systems, distributed networks) either in the state, the control input, or the measurements (see [1]-[5] and the references therein). Time-delay is, in many cases, a source of instability. The stability issue and the performance of linear control systems with delay are, therefore, of theoretical and practical important.

Since the late 1980s, the H_∞ control problem has also attracted much attention due to its both practical and theoretical important. Various approaches have been introduced and a great number of results for continuous systems as well as discrete systems have been investigated in the literature; seem, for instance, [6]-[7]. Very recently, interest has been focused on H_∞ control problem for delay systems. Lee et al. [8] generalized the H_∞ results for continuous systems to systems with state delay, which was further extended to systems with both state and input delays in [9] and [10], respectively. In the context of discrete systems with state delay, similar results can be found in [11] and references therein.

Recently, increasing attention has been paid to the discussion of the theory of neutral delay systems and some issues, such as stability and stabilization, related to such systems have been studied [12]-[17]. To date, however, very little attention has been draw to the problem of robust H_∞ control, for stochastic neutral delay systems with dependence of delay, these are more complex and still open.

In the present paper, we study the robust H_∞ control for stochastic neutral delay systems dependent of delay. Then, we address the robust H_∞ control design problem such that the stabilization of the closed-loop feedback system is guaranteed with a prescribed H_∞-norm bound constraint on disturbance attenuation for all admissible uncertainties. Throughout the paper, the main thrust stems from a Lyapunov-Krasovskii functional approach. In terms of a linear matrix inequality [18], then, a sufficient condition for the existence of H_∞ state feedback controller is presented.

Notation:

Most notations used in this paper are fairly standard. R^n and $R^{n\times m}$ denote the n dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. The A^T denotes the transpose of matrix A. The $\|A\|$ denotes the standard Euclidean norm of matrix A. $\lambda_{\max}(P)$ stands for the operation of taking the maximum eigenvalue of P.

Ⅱ. Preliminaries

Consider the uncertain nonlinear stochastic neutral differential delay equation

$$\begin{aligned}
&d[x(t)+Cx(t-h)]\\
&=[(A+\Delta A(t))x(t)+(A_h+\Delta A_h(t))x(t-h)+F(x(t),x(t-h))+Bu(t)+E_1 v(t)]dt\\
&+[G(x(t),x(t-h))+E_2 v(t)]\,dw(t),
\end{aligned}$$

$$z(t)=Lx(t)+L_h x(t-h), \qquad (4)$$

where $x(t) \in R^n$ is the state vector, $u(t)$ is a control input with appropriate dimensions, $v(t)$ is the noise signal which belongs to $L_2[0,\infty)$ with appropriate dimensions, $w(t)$ is a scalar Brownian motion defined on the probability space $(\Omega, F, \{F_t\}_{t\geq 0}, P)$, $z(t)$ is the controlled output with appropriate dimensions, A, B, C, A_h, E_1 and E_2 are known constant matrices with appropriate dimensions, h are time-invariant bounded delay times satisfying $0 \leq h \leq \bar{h}$. $\Delta A(t)$ and $\Delta A_h(t)$ are unknown matrices representing time-varying parameter uncertainty , and are assumed to be of the form

$$[\Delta A(t) \quad \Delta A_h(t)] = MF(t)[N_1 \quad N_2], \tag{5}$$

Fact 1 (Mao [26])**:** The trivial solution of a neutral stochastic differential equation

$$d[x(t) - G(x(t-h)] = f(t, x(t), x(t-h))dt + g(t, x(t), x(t-h))dw(t), \tag{6}$$

with $f : R_+ \times R^n \times R^n \to R^n$, $g : R_+ \times R^n \times R^n \to R^{n\times m}$ and $G : R^n \to R^n$ sufficiently differentiable maps, is globally asymptotically stable in probability if there exists a function $V(t,x) \in C_2(R_+ \times R^n)$ which is positive definite in the Lyapunov sense, and satisfies

$$\begin{aligned} LV(t,x,y) &= \frac{\partial V(t, x - G(y))}{\partial t} + gradV(t, x - G(y))\, f(t,x,y) \\ &\quad + \frac{1}{2} tr\, g(t,x,y)\, g^T(t,x,y)\, HessV(t, x - G(y)) \leq 0, \end{aligned} \tag{7}$$

for $x \neq 0$. The matrix $HessV$ is the Hessian matrix of the second-order partial derivatives. This fact is analogous to the well-known theorem of Lyapunov for deterministic systems.
Assumption 1 (Lipschitz condition):

(1) $F(0,0) = 0$, $G(0,0) = 0$
(2) $\|F(x_1, x_2) - F(y_1, y_2)\| \leq \|g_1(x_1 - y_1)\| + \|g_2(x_2 - y_2)\|$
(3) $\|G(x_1, x_2) - G(y_1, y_2)\| \leq \|g_3(x_1 - y_1)\| + \|g_4(x_2 - y_2)\|$

for all x_1, x_2, y_1, $y_2 \in R^n$, where g_1, g_2, g_3 and g_4 are known real constant matrices.

Ⅲ. Main Results

In this section, we first state robust stabilization problem for the stochastic time-delay system with $v(t) \equiv 0$ and $u(t) = Kx(t)$. Equation (4) can be written in the following form:

$$\begin{aligned} &d[x(t) + Cx(t-h) + A_h \int_{t-h}^{t} x(s)ds)] \\ &= [(A_f + A_h + \Delta A(t))\, x(t) + \Delta A_h x(t-h) + F(x(t), x(t-h)]dt \\ &+ [G(x(t), x(t-h))]\, dw(t), \end{aligned} \tag{8}$$

where

$A_f = A + BK$.

Define the operator $Z : C([-h = \max\{h_i\}, 0], R^n) \to R^n$ as

$$Z(x(t)) = x(t) + Cx(t-h) + A_h \int_{t-h}^{t} x(s)ds . \qquad (9)$$

We have the following results.

Remark 1: The operator Z is stable if the difference-integral system

$$x(t) + Cx(t-h) + A_h \int_{t-h}^{t} x(s)ds = 0 \qquad (10)$$

is asymptotically stable. Using the terminology in Courtemanche et al. [27], such an equation is known as an integral delay equation.

The stability of (9) is equivalent to the fact that there exists a $\delta > 0$ such that all solutions λ of the associated characteristic equation

$$\det\left[I + C \cdot e^{-sh} + A_i \int_{-h_i}^{0} e^{s\theta} d\theta \right] = 0, \qquad s \in C \qquad (11)$$

satisfy $\mathrm{Re}(\lambda) \le -\delta < 0 \, (\delta > 0)$. A sufficient condition is that the inequality (12) holds

$$C e^{-sh} + A_h \int_{-h}^{0} e^{s\theta} d\theta \le \|C\| + h\|A_h\| < 1 . \qquad (12)$$

Lemma 1: Let $x \in R^n$, $y \in R^m$, $S \in R^{n\times m}$ and $\varepsilon > 0$

$$2 x^T Sy \le \varepsilon x^T x + \varepsilon^{-1} y^T S^T Sy . \qquad (13)$$

3.1 Robust feedback stabilization

Theorem 1:

Given scalar $h > 0$, system (8) is robustly stable in probability, if the operator Z is stable and there exist symmetric positive-definite matrices X, R, Q scalar constants $\alpha > 0$, $\varepsilon_i > 0, i = 1, \cdots, 3$, $\rho > 0$, such that the following LMI holds:

$$\begin{bmatrix} \Omega & I & \Omega_1 & X & X & \Omega_2 & \Omega_3 & M & X N_1^T \\ I & -\varepsilon_1 & 0 & 0 & 0 & 0 & 0 & \rho X C^T M & X N_2^T \\ \Omega_1^T & 0 & -\Sigma_1 & 0 & 0 & 0 & 0 & \rho h X A_h^T & 0 \\ X & 0 & 0 & -R^{-1} & 0 & 0 & 0 & 0 & 0 \\ X & 0 & 0 & 0 & -(hQ)^{-1} & 0 & 0 & 0 & 0 \\ \Omega_2^T & 0 & 0 & 0 & 0 & -\Sigma_2 & 0 & 0 & 0 \\ \Omega_3^T & 0 & 0 & 0 & 0 & 0 & -\Sigma_3 & 0 & 0 \\ M^T & (\rho X C^T M)^T & (\rho h X A_h^T)^T & 0 & 0 & 0 & 0 & -\alpha^{-1} & 0 \\ N_1 X & N_2 X & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha \end{bmatrix} < 0, \tag{13}$$

where

$$\Omega = BY + Y^T B^T + (A + A_h)X + X(A + A_h)^T,$$
$$\Omega_1 = [X g_1^T \;\; X g_3^T], \;\; \Sigma_1 = diag(\frac{1}{2}(\varepsilon_1 + \rho\,\varepsilon_2 + \rho\,\varepsilon_3)^{-1}, \frac{1}{2}(\rho)^{-1}),$$
$$\Omega_2 = \rho X(A + A_h)^T C + \rho Y^T B^T C,$$
$$\Omega_3 = \rho h X(A + A_h)^T A_h + \rho h Y^T B^T A_h,$$
$$\Sigma_2 = R - \rho\,\varepsilon_2^{-1} C^T C - 2(\varepsilon_1 + \rho\,\varepsilon_2 + \rho h\,\varepsilon_3)\, g_2^T g_2 - 2\rho\, g_4^T g_4,$$
$$\Sigma_3 = h(Q - \rho\,\varepsilon_3^{-1} A_h^T A_h).$$

Then, a suitable stabilizing control law is given by $u(t) = Kx(t) = Y X^{-1} x(t)$, $Y \in R^{m\times n}$, where

$$K = Y X^{-1}.$$

Proof:

Let $Z(x(t)) = x(t) + Cx(t-h) + A_h \int_{t-h}^{t} x(s)ds)$ and consider the proposed Lyapunov-Krasovskii type functional Kolmanovskii [29]

$$V = Z^T(x(t))PZ(x(t)) + \int_{t-h}^{t} x^T(\theta)Rx(\theta)d\theta + \int_{-h}^{0}\int_{t+\beta}^{t} x^T(\rho)Rx(\rho)d\rho d\beta. \tag{14}$$

Along trajectories of (8) and making use of the Itô-differential rule Khasminskii [25], then one has the generator LV for the evolution of V as

$$\begin{aligned} LV = {} & x^T(t)Rx(t) - x^T(t-h)Rx(t-h) + h\,x^T(t)Qx(t) - \int_{t-h}^{t} x^T(\beta)Qx(\beta)d\beta \\ & + 2\{x(t) + Cx(t-h) + A_h \int_{t-h}^{t} x(\theta)d\theta]\}^T P\{(A + BK + A_h + \Delta A(t))x(t) \\ & + \Delta A_h(t)x(t-h) + F(x(t), x(t-h)\} \\ & + [G(x(t), x(t-h)]^T P[G(x(t), x(t-h)]. \end{aligned} \tag{15}$$

From (15), one has

$$2x^T(t)P\Delta A(t)x(t)+2x^T(t)P\Delta A_h(t)x(t-h)+2x^T(t-h)C^T P\Delta A(t)x(t)$$
$$+2x^T(t-h)C^T P\Delta A_h x(t-h)+2\int_{t-h}^{t}x^T(\theta)A_h^T P\Delta A(t)x(t)d\theta$$
$$+2\int_{t-h}^{t}x^T(\theta)A_h^T P\Delta A_h(t)x(t)d\theta$$
$$=\frac{1}{h}\int_{t-h}^{t}\begin{bmatrix}x^T(t) & x^T(t-h) & x^T(\theta)\end{bmatrix}\Gamma\begin{bmatrix}x(t)\\ x(t-h)\\ x(\theta)\end{bmatrix}d\theta, \tag{16}$$

where

$$\Gamma=\begin{bmatrix}PM\\ C^T PM\\ hA_h^T PM\end{bmatrix}F(t)\begin{bmatrix}N_1 & N_2 & 0\end{bmatrix}+\begin{bmatrix}N_1^T\\ N_2^T\\ 0\end{bmatrix}F(t)\begin{bmatrix}M^T P & M^T PC & hM^T PA_h\end{bmatrix}.$$

Using Assumption 1, we have

$$\|F(x(t),x(t-h))\|\le\|g_1 x(t)\|+\|g_2 x(t-h)\|, \tag{17}$$

and

$$\|G(x(t),x(t-h))\|\le\|g_3 x(t)\|+\|g_4 x(t-h)\|, \tag{18}$$

hence

$$\|F(x(t),x(t-h))\|^2\le 2\|g_1 x(t)\|^2+2\|g_2 x(t-h)\|^2, \tag{19}$$

and

$$\|G(x(t),x(t-h))\|^2\le 2\|g_3 x(t)\|^2+2\|g_4 x(t-h)\|^2. \tag{20}$$

By Lemma 1 and $P\le\rho I$, we can find that

$$LV\le\frac{1}{h}\int_{t-h}^{t}\begin{bmatrix}x^T(t) & x^T(t-h) & x^T(\theta)\end{bmatrix}(U+\Gamma)\begin{bmatrix}x(t)\\ x(t-h)\\ x(\theta)\end{bmatrix}d\theta, \tag{21}$$

where

$$U = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix},$$

where

$$\phi_{11} = P(A + A_h) + (A + A_h)^T P + PBK + (PBK)^T + \varepsilon_1^{-1} PP$$
$$+ 2(\varepsilon_1 + \rho\varepsilon_2 + \rho h\varepsilon_3) g_1^T g_1 + 2\rho g_3^T g_3 + R + hQ,$$

$$\phi_{12} = \phi_{21}^T = P(A + A_h + BK)^T C,$$

$$\phi_{13} = \phi_{31}^T = \rho h(A + A_h + BK)^T A_h$$

$$\phi_{22} = -R + \rho\varepsilon_2^{-1} C^T C + 2(\varepsilon_1 + \rho\varepsilon_2 + \rho h\varepsilon_3) g_2^T g_2 + 2\rho g_4^T g_4,$$

$$\phi_{23} = \phi_{32}^T = 0, \quad \phi_{33} = -hQ + \rho\varepsilon_3^{-1} h A_h^T A_h.$$

From (4), we obtain

$$U + \Gamma < 0, \tag{22}$$

It follows from the Schur complement that (22) is equivalent to

$$\begin{bmatrix} \Theta & P & \Theta_1 & I & I & \Theta_2 & \Theta_3 & PM & N_1^T \\ P & -\varepsilon_1 & 0 & 0 & 0 & 0 & 0 & \rho C^T M & N_2^T \\ \Theta_1^T & 0 & -\Sigma_1 & 0 & 0 & 0 & 0 & \rho h A_h^T & 0 \\ I & 0 & 0 & -R^{-1} & 0 & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & -(hQ)^{-1} & 0 & 0 & 0 & 0 \\ \Theta_2^T & 0 & 0 & 0 & 0 & -\Sigma_2 & 0 & 0 & 0 \\ \Theta_3^T & 0 & 0 & 0 & 0 & 0 & -\Sigma_3 & 0 & 0 \\ M^T & (\rho C^T M)^T & (\rho h A_h^T)^T & 0 & 0 & 0 & 0 & -\alpha^{-1} & 0 \\ N_1 & N_2 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha \end{bmatrix} < 0, \tag{23}$$

where

$$\Theta = P(A + A_h) + (A + A_h)^T P + PBK + (PBK)^T,$$
$$\Theta_1 = [g_1^T \;\; g_3^T], \quad \Sigma_1 = diag[\frac{1}{2}(\varepsilon_1 + \rho\varepsilon_2 + \rho\varepsilon_3)^{-1}, \frac{1}{2}(\rho)^{-1}],$$
$$\Theta_2 = \rho(A + A_h + BK)^T C, \quad \Theta_3 = \rho h(A + A_h + BK)^T A_h,$$
$$\Sigma_2 = R - \rho\varepsilon_2^{-1} C^T C - 2(\varepsilon_1 + \rho\varepsilon_2 + \rho h\varepsilon_3) g_2^T g_2 - 2\rho g_4^T g_4,$$
$$\Sigma_3 = h(Q - \rho\varepsilon_3^{-1} A_h^T A_h).$$

Pre- and post-multiplying both sides of (23) by $diag(P^{-1}, I, I, I, I, I, I, I, I)$ and denoting $X = P^{-1}$, $Y = KX$ yield (13). We show that (13) guarantees the negativeness, which immediately implies that the closed-loop stochastic neutral nonlinear time delay system (8) is robustly stabilizable.

3.2 Robust H_∞ performance

In this section, we extend the robust stabilization results developed in the previous section to the case of robust H_∞ performance problem, we consider the (4) as above.

Theorem 2:

Given scalar $h > 0$, system (4) is robustly stable in probability, if the operator Z is stable and there exist symmetric positive-definite matrices X, R, Q scalar constants $\alpha > 0$, $\varepsilon_i > 0, i = 1, \cdots, 3$, $\rho > 0$, such that the following LMI holds:

$$\begin{bmatrix} \tilde{\Omega} & I & \tilde{\Omega}_1 & X & X & XL^T & \tilde{\Omega}_2 & \tilde{\Omega}_3 & E_1 & M & XN_1^T \\ I & -\varepsilon_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Pi_1 & XN_2^T \\ \tilde{\Omega}_1^T & 0 & -\tilde{\Sigma}_1 & 0 & 0 & 0 & 0 & 0 & 0 & \Pi_2 & 0 \\ X & 0 & 0 & -R^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ X & 0 & 0 & 0 & -(hQ)^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ LX & 0 & 0 & 0 & 0 & -I & 0 & 0 & 0 & 0 & 0 \\ \tilde{\Omega}_2^T & 0 & 0 & 0 & 0 & 0 & -\tilde{\Sigma}_2 & 0 & \Psi_1 & 0 & 0 \\ \tilde{\Omega}_3^T & 0 & 0 & 0 & 0 & 0 & 0 & -\tilde{\Sigma}_3 & \Psi_2 & 0 & 0 \\ E_1^T & 0 & 0 & 0 & 0 & 0 & \Psi_1^T & \Psi_2^T & -\tilde{\Sigma}_4 & 0 & 0 \\ M^T & \Pi_1^T & \Pi_2^T & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha^{-1} & 0 \\ N_1X & N_2X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha \end{bmatrix} < 0, \qquad (24)$$

where

$$\tilde{\Omega} = BY + Y^T B^T + (A + A_h)X + X(A + A_h)^T,$$

$$\tilde{\Omega}_1 = [X g_1^T \;\; X g_3^T],\; \tilde{\Sigma}_1 = diag[\frac{1}{2}(\varepsilon_1 + \rho\varepsilon_2 + \rho\varepsilon_3)^{-1}, \frac{1}{2}(\rho)^{-1}],$$

$$\tilde{\Omega}_2 = \rho X(A + A_h)^T C + X L^T L_d + \rho Y^T B^T C,$$

$$\tilde{\Omega}_3 = \rho h X(A + A_h)^T A_h + \rho h Y^T B^T A_h,$$

$$\tilde{\Sigma}_2 = R - \rho\varepsilon_2^{-1} C^T C - 2(\varepsilon_1 + \rho\varepsilon_2 + \rho h\varepsilon_3) g_2^T g_2 - L_d^T L_d - 2\rho(1 + \varepsilon_4) g_4^T g_4,$$

$$\tilde{\Sigma}_3 = h(Q - \rho\varepsilon_3^{-1} A_h^T A_h).$$

$$\tilde{\Sigma}_4 = \gamma^2 I - \rho(1 + \varepsilon_4^{-1}) E_2^T E_2, \quad \Pi_1 = \rho X C^T M, \quad \Pi_2 = \rho h X A_h^T, \quad \Psi_1 = \rho C^T E_1,$$

$$\Psi_2 = \rho h A_h^T E_1.$$

Then the state feedback controller

$$u(t) = Kx(t) = Y X^{-1} x(t) \qquad (24)$$

stabilizes system (4) and guarantees that the H_∞ norm bound of the closed-loop has a prescribed level $\gamma > 0$.

Proof: Applying the controller (24) to the (4), we obtain the resulting closed-loop system in the following

$$d[x(t)+Cx(t-h)]$$
$$=[(A_f+\Delta A(t))x(t)+(A_h+\Delta A_h(t))x(t-h)+F(x(t),x(t-h))+E_1 v(t)]dt$$
$$+[G(x(t),x(t-h))+E_2 v(t)]\,dw(t),$$

$$z(t)=Lx(t)+L_h x(t-h), \tag{25}$$

where

$A_f = A + BK$.
For $t > 0$, set

$$J(t)=E\{\int_0^t [z^T(\theta)z(\theta)-\gamma^2 v^T(\theta)v(\theta)]d\theta\}, \tag{26}$$

since

$$E\{V(t)\}=E\{\int_0^t LV(\theta)d\theta\}, \tag{27}$$

therefore, one has

$$J(t)=E\{\int_0^t [z^T(\theta)z(\theta)-\gamma^2 v^T(\theta)v(\theta)+LV(\theta)]d\theta\}-E\{V(t)\}$$
$$\le E\{\int_0^t [z^T(\theta)z(\theta)-\gamma^2 v^T(\theta)v(\theta)+LV(\theta)]d\theta\}$$
$$\le E\{\int_0^t \left[x^T(\theta) \quad x^T(\theta-h) \quad \right]$$

Ⅳ. Examples

Example 1. Consider a linear stochastic neutral interval delay system [30]

$$d[x(t)+\sum_{i=1}^{2}(C_i+\Delta C_i)x(t-h_i)]$$
$$=[A_0 x(t)+\sum_{i=1}^{2}A_i x(t-h_i)]dt+[\Delta Dx(t)+\Delta Ex(t-\tau)]\,dw(t), \tag{26}$$

where

$$A_0=\begin{bmatrix}-2 & 1\\ 0 & -2\end{bmatrix},\ A_1=\begin{bmatrix}1 & 0.05\\ 0.05 & 1\end{bmatrix},\ A_2=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},\ C_1=\begin{bmatrix}0.1 & 0.05\\ 0.05 & 0.1\end{bmatrix},$$

$$C_2=\begin{bmatrix}0.05 & 0\\ 0 & 0.05\end{bmatrix}, \Delta C_i=[-C_{im},C_{im}],\ \mathrm{i}=1,2,\ \Delta D=[-D_m,D_m],\ \Delta E=[-E_m,E_m]$$

$$C_{1m}=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},\ C_{2m}=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},\ D_m=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},\ E_m=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},\ h_1=0.33,$$

$h_2=1.0$. Applying Theorem 1, by the software package LMI Lab., we find the solutions of the LMI as

$$P=\begin{bmatrix}0.3052 & 0.2422\\ 0.2422 & 0.7342\end{bmatrix},\ R_1=\begin{bmatrix}0.1268 & 0.0170\\ 0.0170 & 0.1516\end{bmatrix},\ R_2=\begin{bmatrix}0.1087 & -0.0037\\ -0.0037 & 0.1336\end{bmatrix},$$

$$\theta=0.1167,\ \sigma=0.1167,\ \gamma=18.8752,\ \alpha=4.1492.$$

Therefore, this implies (26) is asymptotically stable in probability for any

$h_1\in[0,0.33]$ and $h_2\in[0,1.0]$.

Example 2. Consider a two–dimensional stochastic neutral interval differential equation [30]

$$d[x(t)+\sum_{i=1}^{2}(C_i+\Delta C_i)x(t-h_i)]$$
$$=[A_0x(t)+\sum_{i=1}^{2}A_ix(t-h_i)+Bu(t)]dt+[\Delta Dx(t)+\Delta Ex(t-\tau)]\,dw(t), \qquad (27)$$

where

$$A_0=\begin{bmatrix}-2 & 1\\ 0 & -2\end{bmatrix},\ A_1=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\ A_2=\begin{bmatrix}0.1 & -0.05\\ 0.05 & 0.1\end{bmatrix},\ B=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix},$$

$$C_1=\begin{bmatrix}0 & 0.2\\ 0.2 & 0\end{bmatrix},\ C_2=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix} C_{1m}=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},\ C_{2m}=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},$$

$$D_m=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},\ E_m=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}.$$

Using Theorem 2 to this uncertain stochastic neutral interval delay equations, it is found that this system is robustly stabilizable for any time-delay $h_1\in[0,0.4]$ and $h_2\in[0,0.89]$ and the correspondent robustly stabilizing control law is

$$u(t)=\begin{bmatrix}-0.0290 & -0.6914\\ -1.1794 & -0.0535\end{bmatrix}x(t)\,.$$

V. Conclusion

This paper has addressed new results and presented insights into the problems of robust stability analysis and robust feedback synthesis for a class of linear neutral interval systems with state delays. The cases of delay-dependence are introduced. Moreover, it has been established that controllers are capable of guaranteeing the closed-loop system stabilization by a linear matrix inequality formulation.

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