行政院國家科學委員會專題研究計畫　成果報告

離散時延不確定性蜂巢式神經網路的觀測器設計

計畫類別：個別型計畫
計畫編號：NSC94-2218-E-018-004-
執行期間：94 年 11 月 01 日至 95 年 07 月 31 日
執行單位：國立彰化師範大學工業教育與技術學系暨研究所

計畫主持人：盧建余

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Design of observer for a class of discrete-time uncertain cellular neural networks with time delay

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一、計畫中英文摘要。

本計劃以離散時延不確定性蜂巢式神經網路來設計其觀測器。互連矩陣與激勵函數被假設為滿足整體 Lipschitz 之條件。參數不確定性被假設為時變有界且位於系統模式矩陣中。我們對本計劃重心為設計一觀測器同時對所有可允許的不確定性都能滿足估測狀態的整體強健漸近穩定。因此我們可將提出此存在的觀測器充份條件轉成線性矩陣不等式法來表示。當我們提出的線性矩陣不等式有解時，所設計的觀測器的正確數學表示式同時也可求得。到目前為止，已發表文獻中還未有考慮強健離散時延蜂巢式神經網路的觀測器設計問題，因此，這領域的探討充滿挑戰。最後本計劃將舉例驗證所提之理論的可應用性。

關鍵詞：觀測器，蜂巢式神經網路，不確定性，線性矩陣不等式

This project is concerned with the problem of observer design for a class of discrete-time uncertain cellular neural networks with time delay. The interconnection matrix and the activation functions are assumed to satisfy the global Lipschitz conditions. The parameter uncertainties are assumed to b time-varying norm-bounded, and appear in all the matrices of the system model. Attention is focused on the design of an observer which ensures global robust stability of the neuron states of estimation for all admissible uncertainties. A sufficient condition for the existence of such an observer is given in terms of a linear matrix inequality (LMI). When this LMI is feasible, the explicit expression of a desired observer is also presented. Up to date, however, no results on design of observer for a class of robust discrete-time cellular neural networks with time delay is available in the literature, this problem is still open and remains challenging. A numerical example is provided to illustrate the applicability of the proposed design approach.

Keywords: observer, cellular neural networks, uncertainty, linear matrix inequality

二、緣由與目的：

1983 Cohen and Grossberg 提出一般性非線性合作競爭式(cooperative-competitive)數學分析模式，稱為 Cohen and Grossberg 神經網路模式。此後，各種類型的神經網路模式(如霍普菲爾神經網路(Hopfield neural networks) ，蜂巢神經網路(Cellular neural networks)及雙向聯想記憶神經網路(bidirectional associative memory neural networks)等等的動態特性如穩定性、存在性及唯一性受到許多的注意及成功地提出各種研究文獻。

於生物與類神經網路中，資訊儲存與傳輸過程中會引發時延現象，如在類比式神經網路電子電路中，由於放大器的交換速度有限，因此在神經元的通訊與反應中會發生時延現象，這種時延現象會影響整個網路上的穩定性，甚至造成系統的振盪或不穩定。

延遲蜂巢式神經網路目前已廣泛地被使用，因此於許多應用中它的神經元狀態估計問題變的非常重要。主要理由為在超大型神經網路中，在整個網路輸出中，經常僅有部份神經元狀態是可獲得。因此為了使神經網路有實際效用，所以經由可獲取的量測來設計估測神經元狀態的觀測器，然後使用已被估測的神經元狀態以獲取確定的實際性能。如系統模式、訊號處理及控制工程。狀態估測是一個很實務的課題且理論的重要性在最近數年也受到很多學者的重視。

本研究計劃是以處理離散時延不確定性蜂巢式神經網路的觀測器設計。而互連矩陣及激勵函數為有界且滿足整體 Lipschitz 條件。互連矩陣及激勵函數出現在系統中。參數不確定性為時變有界同時都落於系統模型矩陣中。我們提出的計劃是經由可獲得量測輸出去設計觀測器使得觀測狀態誤差動態特性是強健整體穩定並且滿足所有可允許的不確定性。利用線性矩陣不等式法求得可解決的充分條件，然後使用凸集最佳化演算法來解線性矩陣不等式條件，並求得我們所設計的觀測器。值得點出線性矩陣不等式的優點，為使用線性矩陣不等式法求解時，雖然有許多參數及矩陣需要被決定，但靠著凸集最佳化演算法可以很有效且快速的求得我們所需要的解。最後我們提出一個數值範例，來說明我們所發展的理論是可用的。

三、主要結果

1. Preliminaries

Consider the following discrete uncertain cellular neural network with a constant delay described by non-linear differential equations of the form

$$x(k+1) = -x(k) + (A + \Delta A(k)) f(x(k)) + (A_d + \Delta A_d(k)) f(x(k-\tau)) + V\,, \qquad (1)$$

where

$x(k) = [x_1(k), x_2(k), \cdots, x_n(k)]^T \in R^n$ denotes the state vector of the cellular neural network and n is the number of neurons, $f(x(k)) = [f_1(x_1(k)), f_2(x_2(k)), \cdots, f_n(x_n(k))]^T$ is the neuron activation function, $\tau \ge 0$ represents the transmission delay which satisfies $0 \le \tau \le \bar{\tau}$, $A = \{a_{ij}\}$ and $A_d = \{a_{ij}^{\ d}\}$ are the interconnection matrices representing the weight coefficients of the neurons, and $V = [V_1, V_2, \cdots, V_n]^T$ is the constant external input vector.

$\Delta A(k)$ and $\Delta A_d(k)$ are unknown matrices representing time-varying parameter uncertainties, and are assumed to be of the form

$$[\Delta A(k)\ \ \Delta A_d(k)] = DF(k)[E_1\ \ E_2]\,, \qquad (2)$$

where D, E_1 and E_2 are known real constant matrices and $F(\cdot): R \rightarrow R^{k \times l}$ is an unknown real-valued time-varying matrix satisfying

$$F^T(k)F(k) \le I\,,\ \ \forall t\,. \qquad (3)$$

Assume further that all the elements of $F(k)$ are Lebesgue measurable. $\Delta A(k)$ and $\Delta A_d(k)$ are said to be admissible if both Eqs. (2) and (3) hold.

Throughout this paper it is assumed that the activation function satisfies the following assumption.

Assumption 1 (Lipschitz condition)**:**
There exist positive constants α_i, $i = 1, 2, \cdots, n$ such that

$$\left| f_i(\zeta_1) - f_i(\zeta_2) \right| \le \alpha_i \left| \zeta_1 - \zeta_2 \right|,\ \ \zeta_1 \ne \zeta_2\,,\ \ i = 1, \cdots, n\,, \qquad (4)$$

for all ζ_1 and ζ_2 $\in R$.

In fact, the information about the neuron states is incomplete from the network measurements (outputs). That is, only partial information about the neuron states is available in the network measurements. In other words, the network measurements are subject to nonlinear disturbances. Accordingly, an effective observer algorithm is developed in order to observe the neuron states from the available network outputs. The neural network measurements are as

$$y(k) = Cx(k) + Gg(x(k)), \tag{5}$$

where $y(k) \in R^m$ is the measurement output, C and G are known real constant matrices, $g: R^n \to R^m$ is known nonlinear function and satisfies the following Lipschitz condition

$$\|g(x_1) - g(x_2)\| \leq \|S_g(x_1 - x_2)\| \tag{6}$$

and hence

$$\|g(x_1) - g(x_2)\|^2 \leq 2\|S_g(x_1 - x_2)\|^2, \tag{7}$$

where $S_g \in R^{n\times n}$ is known real constant matrix.

The objective of the present analysis is to design a state observer such that the error dynamics is globally asymptotically stable for all admissible uncertainties and the addressed nonlinearity. More specifically, the observer design is of the form

$$\hat{x}(k+1) = -\hat{x}(k) + Af(\hat{x}(k)) + A_d f(\hat{x}(k-\tau)) + V + L[y(k) - C\hat{x}(k) - Gg(\hat{x}(k))] \tag{8}$$

such that the error dynamics is globally asymptotical stability, where the observer gain L is to be determined, $\hat{x}(k)$ is the estimation of the neuron state.

Define the error state

$$e(k) = x(k) - \hat{x}(k). \tag{9}$$

From (1), (5) and the observer (8), it is easy to show that

$$e(k+1) = (LC - I)e(k) + A\left[f(x(k)) - f(\hat{x}(k))\right] + \Delta A(k) f(x(k))$$

$$+ A_d\left[f(x(k-\tau)) - f(\hat{x}(k-\tau))\right] + \Delta A_d\, f(x(k-\tau)) + G(g(x(k)) - g(\hat{x}(k))). \tag{10}$$

The main purpose is to design a state observer L such that the error dynamics remain globally asymptotically stable for all admissible uncertainties and the nonlinear disturbance.

3. Main Results

This section explores the globally robust stability of error dynamics given in (10). Specially, an LMI approach is employed to solve the robust stability if the system in (10) is globally asymptotically stable for all admissible uncertainties $\Delta A(k)$ and $\Delta A_d(k)$. The analysis commences by using the LMI approach to develop some results which are essential to introduce the following Lemma 1 for the development of our main theorem.

Lemma 1: Let D and S be real matrices of appropriate dimensions. Then the following statements hold for vectors $x,\ y \in R^n$

$$2x^T DSy \le x^T D D^T x + y^T S^T S\, y. \tag{11}$$

To study the globally robust stability of error dynamics, the following theorem reveals that such conditions can be expressed in terms of LMIs.

Theorem 1: Consider the discrete uncertain delayed cellular neural network (1) satisfying Assumption 1. If there exist matrices P and Q such that the following LMI holds

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} & \Omega_{17} & \Omega_{18} \\ \Omega_{12}^T & -P & 0 & 0 & 0 & 0 & 0 & 0 \\ \Omega_{13}^T & 0 & -I & 0 & 0 & 0 & 0 & 0 \\ \Omega_{14}^T & 0 & 0 & -P & 0 & 0 & 0 & 0 \\ \Omega_{15}^T & 0 & 0 & 0 & -I & 0 & 0 & 0 \\ \Omega_{16}^T & 0 & 0 & 0 & 0 & -I & 0 & 0 \\ \Omega_{17}^T & 0 & 0 & 0 & 0 & 0 & -I & 0 \\ \Omega_{18}^T & 0 & 0 & 0 & 0 & 0 & 0 & -P \end{bmatrix} < 0 \tag{12}$$

, then a robust observer is given by (8) with $L = P^{-1}Y$, where

$\Omega_{11} = -P + Q$, $\Omega_{12} = \sqrt{3}(C^T Y^T - I)$, $\Omega_{13} = (C^T Y^T - I)M$, $\Omega_{14} = 2S_g^T G^T Y^T$,

$\Omega_{15} = S_g^T G^T Y^T$, $\Omega_{16} = S_g^T G^T Y^T M$, $\Omega_{17} = 2\Gamma_1^T A^T PM$, $\Omega_{18} = 2\sqrt{2}\,\Gamma_1^T A^T P$,

where $\Gamma_1 = diag(\alpha_{11}, \alpha_{12}, \alpha_{13}, \cdots, \alpha_{1n}) > 0$ and S_g is known real constant matrix.

Proof: Consider the following Lyapunov function candidate for the system in (10)

$$V_k = e(k)^T Pe(k) + \sum_{i=1}^{\tau} e(k-i)^T Qe(k-i). \tag{13}$$

Then the difference between the Lyapunov function candidates for two consecutive time instants is as follows

$$\begin{aligned}
\Delta V_k &= V(e(k+1)) - V(e(k)) \\
&= e^T(k)((LC-I)^T P(LC-I) - P + Q)e(k) - e^T(k-\tau)Qe(k-\tau) \\
&+ 2e^T(k)(LC-I)^T P\Delta A(k) f(x(k)) + 2e^T(k)(LC-I)^T P\Delta A_d(k) f(x(k-\tau)) \\
&+ 2e^T(k)(LC-I)^T P[A(f(x(k)) - f(\hat{x}(k))) + A_d(f(x(k-\tau)) - f(\hat{x}(k-\tau)))] \\
&+ 2e^T(k)(LC-I)^T P(g(x(k)) - g(\hat{x}(k))) + f^T(x(k))\Delta A^T(k) P\Delta A(k) f(x(k)) \\
&+ 2f^T(x(k))\Delta A^T(k) P\Delta A_d(k) f(x(k-\tau)) \\
&+ 2f^T(x(k))\Delta A^T(k) P\ [A(f(x(k)) - f(\hat{x}(k))) + A_d(f(x(k-\tau)) - f(\hat{x}(k-\tau)))] \\
&+ 2f^T(x(k))\Delta A^T(k) PLG(g(x(k)) - g(\hat{x}(k))) \\
&+ f^T(x(k-\tau))\Delta A_d^T(k) P\Delta A_d(k) f(x(k-\tau)) \\
&+ 2f^T(x(k-\tau))\Delta A_d^T(k) P\ [A(f(x(k)) - f(\hat{x}(k))) + A_d(f(x(k-\tau)) - f(\hat{x}(k-\tau)))] \\
&+ 2f^T(x(k-\tau))\Delta A_d^T(k) PLG(g(x(k)) - g(\hat{x}(k)))
\end{aligned}$$

$$+[A(f(x(k))-f(\hat{x}(k)))+A_d(f(x(k-\tau))-f(\hat{x}(k-\tau)))]^T P$$
$$\cdot[A(f(x(k))-f(\hat{x}(k)))+A_d(f(x(k-\tau))-f(\hat{x}(k-\tau)))]$$
$$+2[A(f(x(k))-f(\hat{x}(k)))+A_d(f(x(k-\tau))-f(\hat{x}(k-\tau)))]^T$$
$$\cdot PLG(g(x(k))-g(\hat{x}(k)))$$
$$+(g(x(k))-g(\hat{x}(k)))^T G^T L^T PLG(g(x(k))-g(\hat{x}(k))). \qquad (14)$$

Accordingly, it follows from Lemma 1 and Assumption 1

$$2e^T(k)(LC-I)^T P[A(f(x(k))-f(\hat{x}(k)))+A_d(f(x(k-\tau))-f(\hat{x}(k-\tau)))]$$
$$\le e^T(k)(LC-I)^T P(LC-I)e(k)$$
$$+2(e^T(k)\Gamma_1^T A^T PA\Gamma_1 e(k)+e^T(k-\tau)\Gamma_2^T A_d^T P A_d \Gamma_2 e(k-\tau)), \qquad (15)$$

where $\Gamma_1 = diag(\alpha_{11}, \alpha_{12}, \alpha_{13}, \cdots, \alpha_{1n})$ and $\Gamma_2 = diag(\alpha_{21}, \alpha_{22}, \alpha_{23}, \cdots, \alpha_{2n})$ and $\alpha_{1i} > 0$ and $\alpha_{2i} > 0$, $i = 1, 2, 3, \cdots, n$ are given in Assumption 1,

$$2e^T(k)(LC-I)^T P(g(x(k))-g(\hat{x}(k)))$$
$$\le e^T(k)(LC-I)^T P(LC-I)e(k)+e^T(k)S_g^T G^T L^T PLG S_g e(k), \qquad (16)$$

where S_g is known real constant matrix,

$$2f^T(x(k))\Delta A^T(k)P[A(f(x(k))-f(\hat{x}(k)))+A_d(f(x(k-\tau))-f(\hat{x}(k-\tau)))]$$
$$\le f^T(x(k))N_1^T N_1 f(x(k))$$
$$+2(e^T(k)\Gamma_1^T A^T PM M^T PA\Gamma_1 e(k)+e^T(k-\tau)\Gamma_2^T A_d^T PM M^T P A_d \Gamma_2 e(k-\tau)), \qquad (17)$$

$$2\, f^T(x(k))\Delta A^T(k)PLG(g(x(k))-g(\hat{x}(k)))$$

$$\leq f^T(x(k))\, N_1^T N_1\, f(x(k)) + e^T(k)\, S_g^T G^T L^T PPLG\, S_g\, e(k)\,, \qquad (18)$$

$$2\, f^T(x(k-\tau))\Delta A_d^T(k)P\ \left[A(f(x(k))-f(\hat{x}(k)))+A_d(f(x(k-\tau))-f(\hat{x}(k-\tau)))\right]$$

$$\leq f^T(x(k-\tau))\, N_2^T N_2\, f(x(k-\tau))$$
$$+2(e^T(k)\Gamma_1^T A^T PM\, M^T PA\Gamma_1 e(k) + e^T(k-\tau)\Gamma_2^T A_d^T PM\, M^T P\, A_d\Gamma_2 e(k-\tau)))\,, \qquad (19)$$

$$2\, f^T(x(k-\tau))\Delta A_d^T(k)PLG(g(x(k))-g(\hat{x}(k)))$$

$$\leq f^T(x(k-\tau))\, N_1^T N_1\, f(x(k-\tau)) + e^T(k)\, S_g^T G^T L^T PM\, M^T PLG\, S_g\, e(k)\,, \qquad (20)$$

$$2\left[A(f(x(k))-f(\hat{x}(k)))+A_d(f(x(k-\tau))-f(\hat{x}(k-\tau)))\right]^T$$
$$\cdot PLG(g(x(k))-g(\hat{x}(k)))$$

$$\leq 2(e^T(k)\Gamma_1^T A^T PA\Gamma_1 e(k) + e^T(k-\tau)\Gamma_2^T A_d^T P\, A_d\Gamma_2 e(k-\tau))$$
$$+ e^T(k)\, S_g^T G^T L^T PLG\, S_g\, e(k)\,, \qquad (21)$$

$$2\left[A(f(x(k))-f(\hat{x}(k)))+A_d(f(x(k-\tau))-f(\hat{x}(k-\tau)))\right]^T P$$
$$\cdot\left[A(f(x(k))-f(\hat{x}(k)))+A_d(f(x(k-\tau))-f(\hat{x}(k-\tau)))\right]$$

$$\leq 4(e^T(k)\Gamma_1^T A^T PA\Gamma_1 e(k) + e^T(k-\tau)\Gamma_2^T A_d^T P\, A_d\Gamma_2 e(k-\tau))\,, \qquad (22)$$

$$2(g(x(k))-g(\hat{x}(k)))^T G^T L^T PLG(g(x(k))-g(\hat{x}(k))$$

$$\leq 2\, e^T(k)\, S_g^T G^T L^T PLG\, S_g\, e(k)\,, \qquad (23)$$

$$2e^T(k)(LC-I)^T P\Delta A(k) f(x(k)) + 2e^T(k)(LC-I)^T P\Delta A_d(k) f(x(k-\tau))$$

$$= 2e^T(k)(LC-I)^T PMF(k)[N_1 \quad N_2][f^T(x(k)) \quad f^T(x(k-\tau))]^T$$

$$\le e^T(k)(LC-I)^T PM M^T P(LC-I)e(k)$$

$$+[N_1 f(x(k)) + N_2 f(x(k-\tau))]^T \cdot [N_1 f(x(k)) + N_2 f(x(k-\tau))], \tag{24}$$

$$f^T(x(k))\Delta A^T(k)P\Delta A(k) f(x(k)) + 2f^T(x(k))\Delta A^T(k)P\Delta A_d(k) f(x(k-\tau))$$

$$+ f^T(x(k-\tau))\Delta A_d^T(k)P\Delta A_d(k) f(x(k-\tau))$$

$$= \begin{bmatrix} f^T(x(k)) & f^T(x(k-\tau)) \end{bmatrix} \begin{bmatrix} \Delta A^T(k) \\ \Delta A_d^T(k) \end{bmatrix} P \begin{bmatrix} \Delta A(k) & \Delta A_d(k) \end{bmatrix} \begin{bmatrix} f(x(k)) \\ f(x(k-\tau)) \end{bmatrix}$$

$$\le \begin{bmatrix} f^T(x(k)) & f^T(x(k-\tau)) \end{bmatrix} \begin{bmatrix} N_1^T \\ N_2^T \end{bmatrix} M^T PM \begin{bmatrix} N_1 & N_2 \end{bmatrix} \begin{bmatrix} f(x(k)) \\ f(x(k-\tau)) \end{bmatrix}, \tag{25}$$

Substituting (15)-(25) into (14), it can be shown that (14) reduces to

$$\Delta V_k \le \Pi^T(k)\Sigma\Pi(k), \tag{26}$$

where

$$\Pi(k) = [e^T(k) \quad e^T(k-\tau) \quad f^T(x(k)) \quad f^T(x(k-\tau))]^T,$$

$$\Sigma = \begin{bmatrix} \Sigma_{11} & 0 & 0 & 0 \\ 0 & \Sigma_{22} & 0 & 0 \\ 0 & 0 & \Sigma_{33} & \Sigma_{34} \\ 0 & 0 & \Sigma_{34}^T & \Sigma_{44} \end{bmatrix}, \tag{27}$$

where

$$\Sigma_{11} = 3(LC-I)^T P(LC-I) - P + Q + 8\Gamma_1^T A^T PA\Gamma_1 + 4\Gamma_1^T A^T PM M^T PA\Gamma_1$$

$$+ S_g^T G^T L^T PM M^T PLG S_g + 4S_g^T G^T L^T PLG S_g + S_g^T G^T L^T PPLG S_g$$

$$+ (LC - I)^T PM M^T P(LC - I),$$

$$\Sigma_{22} = -Q + 8\Gamma_2^T A_d^T P A_d \Gamma_2 + 4\Gamma_2^T A_d PM M^T P A_d^T \Gamma_2,$$

$$\Sigma_{33} = N_1^T M^T PM N_1 + 8 N_1^T N_1, \quad \Sigma_{34} = N_1^T M^T PM N_2 + 8 N_1^T N_2,$$

$$\Sigma_{44} = N_2^T M^T PM N_2 + 8 N_2^T N_2.$$

By the Schur complement, (26) is equal to

$$\Sigma = \begin{bmatrix} \Sigma_{11} & 0 & 0 \\ 0 & \Sigma_{22} & 0 \\ 0 & 0 & \Sigma_{33} - \Sigma_{34}\Sigma_{44}^{-1}\Sigma_{34}^T \end{bmatrix}. \tag{28}$$

On the other hand, error dynamics (10) is asymptotically stable for all admissible uncertainties which implies $\Sigma < 0$, and hence $\Delta V_k < 0$.

Eq. (28) yields $\Sigma_{11} < 0$, $\Sigma_{22} < 0$ and $\Sigma_{33} - \Sigma_{34}\Sigma_{44}^{-1}\Sigma_{34}^T < 0$. Therefore, by the Schur complement, $\Sigma_{11} < 0$ guarantees (12). Therefore, it can be concluded from the Lyapunov-Krasovskii functional that the error dynamics expressed in (10) attains globally asymptotically robust stability. This completes the proof of Theorem 1.

Two numerical examples are now presented to demonstrate the usefulness of the proposed approach.

IV. Examples

Example 1: Consider the following discrete uncertain delayed cellular neural network

$$x(k+1) = -x(t) + (A + \Delta A(k)) f(x(k)) + (A_d + \Delta A_d(k)) f(x(k-\tau)) + V, \tag{29}$$

$$y(k) = Cx(k) + Gg(x(k)), \tag{30}$$

where

$$A = \begin{bmatrix} -1 & 0 \\ -0.5 & -1 \end{bmatrix}, \quad A_d = \begin{bmatrix} 0.9 & -1.2 \\ 0.05 & -0.9 \end{bmatrix}, \quad C = \begin{bmatrix} 0.3 & 0.2 \\ 0.5 & 0.4 \end{bmatrix}, \quad G = \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.3 \end{bmatrix},$$

$$D=\begin{bmatrix}0.1 & 0\\ 0 & 0.3\end{bmatrix},\ E_1=\begin{bmatrix}0.02 & 0.02\\ -0.1 & 0.2\end{bmatrix},\ E_2=\begin{bmatrix}-0.07 & 0.3\\ -0.01 & 0.2\end{bmatrix},$$

$$\Gamma_1=\begin{bmatrix}0.1 & 0\\ 0 & 0.2\end{bmatrix},\ S_g=\begin{bmatrix}0 & 0.3\\ 0.1 & 0.2\end{bmatrix},\ V=[1 \quad 1]^T.$$

In this example, the time delay is assumed $\tau=1$. Utilizing Theorem 1, the robust observer design is considered. By resorting to the Matlab LMI Control Toolbox and solving the LMI (12), it is found that the solution is as follows

$$P=\begin{bmatrix}4.6797 & 0.2715\\ 0.2715 & 4.0060\end{bmatrix},\ Q=\begin{bmatrix}1.9242 & -0.0498\\ -0.0498 & 1.1568\end{bmatrix},\ Y=\begin{bmatrix}3.1306 & -0.0167\\ -0.0167 & 2.3260\end{bmatrix}.$$

Therefore, by Theorem 1 the robust design problem is solvable and a desired observer is given as $L=P^{-1}Y=\begin{bmatrix}0.8362\\ 0.4250\end{bmatrix}$.

Example 2: Consider the discrete uncertain delayed cellular neural network (29) and (30) with parameters as follows

$$A=\begin{bmatrix}0.05 & 0.25 & 0.05\\ 0.10 & 0.05 & 0.15\\ 0.15 & 0.15 & 0.05\end{bmatrix},\ A_d=\begin{bmatrix}0.75 & 0.75 & 0.95\\ 0 & 0.50 & 0.75\\ 0.15 & 0.95 & 0.95\end{bmatrix},\ C=\begin{bmatrix}0.3 & 0.2 & 0\\ 0 & 0.2 & 0.1\\ 0.2 & 0.1 & 0.2\end{bmatrix},$$

$$G=\begin{bmatrix}0.2 & 0.1 & 0.2\\ 0.1 & 0.2 & 0\\ 0 & 0.2 & 0.1\end{bmatrix},D=\begin{bmatrix}0.1 & 0 & 0.1\\ 0 & 0.3 & 0.1\\ 0 & 0.1 & 0.2\end{bmatrix},\ E_1=\begin{bmatrix}0.02 & 0.02 & 0\\ -0.1 & 0.1 & 0.1\\ 0.1 & 0 & 0.1\end{bmatrix},$$

$$E_2=\begin{bmatrix}0.02 & -0.02 & 0\\ 0.1 & -0.1 & 0\\ 0.1 & 0 & 0.1\end{bmatrix},\ \Gamma_1=\begin{bmatrix}0.1 & 0 & 0\\ 0 & 0.2 & 0\\ 0 & 0 & 0.3\end{bmatrix},\ S_g=\begin{bmatrix}0.1 & 0.2 & 0.1\\ 0.1 & 0.2 & 0.2\\ 0.1 & 0.3 & 0.2\end{bmatrix},$$

$$\tau=2,\ V=[1 \quad 1 \quad 1]^T.$$

Utilizing Theorem 1, the robust observer design is considered. By resorting to the

Matlab LMI Control Toolbox and solving the LMI (12), the corresponding solution is found as follows

$$P=\begin{bmatrix} 2.0633 & -0.1040 & 0.1169 \\ -0.1040 & 2.2178 & -0.0988 \\ 0.1169 & -0.0988 & 2.1552 \end{bmatrix},\quad Q=\begin{bmatrix} 0.7819 & 0.0220 & 0.1238 \\ 0.0220 & 0.6672 & 0.1512 \\ 0.1238 & 0.1512 & 0.5604 \end{bmatrix},$$

$$Y=\begin{bmatrix} 2.1411 & -0.1219 & -0.5484 \\ -0.1219 & 2.3370 & 0.0817 \\ -0.5484 & 0.0817 & 2.0990 \end{bmatrix}.$$

Therefore, by Theorem 1 the robust design problem is solvable and a desired observer is given as $L=P^{-1}Y=\begin{bmatrix} 0.7454 \\ 1.1215 \\ 1.2744 \end{bmatrix}$.

四、討論

This report has investigated the problem of observer design for a particular class of discrete uncertain delayed neural network. A sufficient condition for the solvability of this problem, which guarantees the asymptotic stability of the error dynamics, has been derived using the Lyapunov-Krasovskii functional and the LMI approach. Two numerical examples have been presented to demonstrate the effectiveness of the proposed approach. It should be pointed out that the solvability conditions provided in this paper are delay-independent, which may be conservative; one way to reduce the conservatism is to develop delay-dependent conditions, which could be a future research topics.

五、計畫成果自評

本計畫已達成預期目標，提出一些頗具學術及實用價值的方法

1. This project addresses the problem of observer design in discrete uncertain delayed cellular neural networks.
2. Two numerical examples are now presented to demonstrate the usefulness of the proposed approach.