OPTIMAL R&D PROJECT VALUATIONS UNDER INDUSTRY COMPETITIONS AND INCOMPLETET INFORMATION

Shian-Chang Huang
Department of Business Administration, National Changhua University of Education,
Changhua, No.2, Shi-Da Road, Changhua 500, Taiwan.
E-mail: shhuang@cc.ncue.edu.tw; Tel: +886-4-7232105-7420

ABSTRACT

The valuation of R&D project involving a number of interdisciplinary knowledge has become an important research field of service science. Traditional researches on R&D project valuations do not consider the influences of technological progress and gradual information revelations on R&D investments. This study develops a framework for optimal valuations of R&D projects under industry competition and incomplete information, which is not considered in traditional real option literature. This study also includes manager's risk preference in considering the optimal exercise policy, which has substantial impacts on the investment timing. The results of this study provide a deeper understanding of the influences of various uncertainties on R&D project valuations and decision makings. The explicit formulas for the optimal valuation and exercise policy of an R&D project provided by this study help managers making good decisions on their investments.

Keywords: R&D valuation; real option; risk sensitive effect; industry competition; incomplete information

1. INTRODUCTION

Rercently, services science applies insights from scientific, management, and engineering perspectives to analyze how to align people and technology effectively to generate value for both services providers and clients. R&D project investments are risky and knowledge-intensive. Its analysis involves a number of interdisciplinary knowledge, and becomes an important research field of service science.

The main financial objective of a company is to maximize shareholders value. This can be achieved through effective investment decisions that imply the financing of the best R&D projects. All companies perform a certain type of quantitative analysis in order to identify value-increasing R&D projects and to formulate their preferences in financing decisions. Traditional real option approach does not consider the influences of technological progress and gradual information revelation on R&D investments, and thus can not produce a fair valuation about a R&D project. Therefore this research focuses on the specific situations usually encountered in R&D investments by considering the industry competition and incomplete information about the future cash flow.

The real-option approach focuses on analyzing investment under uncertainty. Namely many investment and operating decisions can be structured in terms of the options they create. For example, the opportunity to invest in a project or to expand production facilities is like a call option on the investment opportunity. The underlying asset is the present value of the net operating cash flows from the investment. The exercise price is the investment cost. The option is exercised by making the investment expenditure. Once the real options embedded in investment opportunities are identified, the valuation models of financial options can be used for real investment analysis.

One difference between our research and traditional real option approaches is that managers holding investment options to exercise have different attitudes toward risk. That is someone may be risk-averse, while someone else may be risk-seeking. So, the same R&D project will be evaluated differently in the managers. Most of the traditional R&D project valuation literature (see Dixit and Pindyck (1994), and Trigeorgis (1996), or recently, Schwartz and Moon (2000a), Cortazar, et al. (2001), Schwartz and Zozaya (2001)) or internet companies valuation researches (see Schwartz and Moon (2000b, 2001)) have examined the value of flexibility in investment and operating decisions, but little has been written about management's attitude toward risk. In this article, it's the risk sensitive effects, a key factor in decision-making for managers. It influences the evaluations of different executives. We can also interpret it as the utility-based pricing method.

The second effect considered is industry competition in R&D investments. New innovation reduces the cost of the firm's investment. This is because technological innovation causes productions to become more efficient, and thus making the new technology investment cheaper. Firms that have adopted to the old technology face tough competition from new entrants and their revenue flow is thus reduced dramatically. Consequently, this study considers the competitive interactions among firms in a given industry. Related literature about industry competition and technology investment like Alvarez and Stenbacka (2001, 2002) consider technological uncertainty alongside revenue uncertainty, but they concern only events that occur after the irreversible investment has been undertaken. This study is interested in exogenous technological progress and competition, which the investor already observes before undertaking the project. For simplicity, we assume that with each competitor entering the market, the project value of the firm is reduced by an uncertain percentage, modeled as a negative exponential jump. Therefore the impact of competition is stochastic. Another randomness for the firm is the timing when competitors choose to exercise their investment option, such a situation is conveniently modeled by a Poisson process which takes account of the random points in time when the gross project value drops. The question that arises now is what optimal strategy the firm should pursue when it faces these random competitive entries.

The main consideration of this article is the information learning effect. That is, managers may acquire information from market data in order to evaluate the new R&D project more precisely. Majd and Pindyck (1989), and Pennings and Lint (1997) examine the intangible asset valuation with learning effect, while Epstein, et al. (1999) use a filtering approach towards learning. The importance of learning actions, like exploration, experimentation was recognized early in the economics literature (e.g., Roberts and Weitzman, 1981). This study introduces an incomplete information framework into the R&D project valuation. A theory on the optimal valuation under incomplete information is developed in this article. Central to the theory is the determination of the dynamics for the best estimate of noisy market information, and the optimal stopping problems based on the filtered information. The dynamics of the value estimated are shown to differ from the dynamics of the true asset value, the optimal timing of investment in these R&D projects and their related problems are also developed in this study.

The present paper is organized as follows: section 2 presents a basic modeling of R&D project valuation under risk sensitive effect and industry competition. Section 3 introduces optimal timing of investment under incomplete information, and adopt a learning framework to derive a value for the R&D project. The last section concludes the paper.

2. GENERAL PROBLEM

In the literature of irreversible investment under uncertainty, a typical investment valuation problem is characterized by the following three features. First, the value of the asset obtained with the investment is a stochastic process; second, the investment decision is irreversible; and third, the investor is free to choose the timing of the investment. In such a setting, choosing the timing of the investment is technically an optimal stopping problem. In this section, we will first introduce the basic ideas of such a problem.

2.1 Optimal Valuation and Timing of Investment

Our study assume the payoff of the manager upon exercise of the investment opportunity is $X_t - K$, where X_t is the stochastic revenue at time t, K the exercise price. The value of the investment opportunity is the expected present value of payoff obtained by optimally exercising the investment opportunity. To this end, the discounted payoff for stopping at time τ is

$$g(t, X_\tau) = e^{-r\tau}(X_\tau - K)^+, \tag{1}$$

where r is a positive discount rate. The parameter r captures the investor's opportunity cost of a delay in exercise. It may also be interpreted as an impatience parameter that captures the investor's preference for future over immediate consumption. In terms of equation (1), the value of an investment can now be defined as the supremum of expected discounted payoff

$$\sup_{\tau\in\Upsilon} E^x\left[g(\tau, X_\tau)\right],$$

where the supremum is computed over the domain $\{\Upsilon\}$ of admissible investment timing.

At each instant time, the only change is the current project (or intangible asset) price. Hence the value function $V(\cdot)$ is a function of asset price. The random variable X_t, captures the state of the R&D asset price process at time t. The value function in state x is given by

$$V(x, K) = \sup_{\tau\in\Upsilon} E^x\left[g(\tau, X_\tau)\right] = E^x\left[g(\tau^*, X_{\tau^*})\right], \tag{2}$$

where τ^* is an optimal stopping time. We solve this optimal stopping problem in the next section in order to arrive at an optimal exercise policy and the value of an R&D intangible asset.

On the other hand, if the investor pays K_t to obtain the project, where K_t is a time-decay function which depends on technological innovations, the payoff of the investment is thus $g(X, I_t) = X - K_t$. The value of the option to invest is:

$$V(x, K_\tau) = \sup_{\tau \in \Upsilon} E^x \left[e^{-r\tau} \left(X - K_\tau \right) \right], \tag{3}$$

where x refers to the current asset value and K_τ the investment cost at some future time τ.

2.2 Optimal Investment Threshold and Project Value

From real option literature, dynamic programming (Bellman, 1957, Bertsekas, 2000), or optimal stopping theorem (Oksendal (2000) chapter 10) that value function V must satisfy the following ordinary differential equation (ODE):

$$-rV + \mathrm{A}V = 0,$$

where A is the characteristic operator of X_t. It is equivalent to the Bellman equation (Bellman, 1957) of optimality in the continuation region:

$$rV(X)dt = E(dV(X)) \ when \ X \in [0, X_{\tau^*})$$

in addition, value function V must satisfy the boundary condition: below the optimal exercise barrier, the value from continuing should exceed the reward from stopping immediately, whereas at or above the barrier, the value is the same as the immediate reward. The boundary condition could be written as

$$V(s, X_{\tau^*}) = e^{-rs}(X_{\tau^*} - K). \tag{4}$$

The next section solves the problem in which the manager is risk sensitive and finds his/her optimal exercise threshold $X^* = X_{\tau^*}$.

2.3 Risk Sensitive Effects and Utility-Based Valuation

In this section considers the decision making of a firm's manager. We assume that he is risk averse and has a negative exponential utility. Using the exponential utility to evaluate investment projects can capture risk sensitivities of a decision maker. If α is the Constant Absolute Risk Aversion parameter (assumed to be greater than zero), the typical form of this utility function is $U(w) = -e^{-\alpha w}$; but we introduce an offset of 1. Thus an option exercise gives rise to $1 - e^{-\alpha(X_\tau - K)}$ as the utility of payoff upon exercise. The offset of 1 makes the utility of zero-payoff become zero and simplifies the problem setting.

First we define the reward for stopping at time t as the discounted utility of terminal payoff

$$U(t, X_t) = e^{-rt}\left(1 - e^{-\alpha(X_t - K)}\right), \tag{5}$$

where

$$\frac{dX_t}{X_t} = \mu dt + \sigma dB_t \tag{6}$$

is the R&D asset value process. In the continuation region, the utility function $U(\cdot)$ of the solution must satisfy the following ODE

$$\mu x \frac{dU}{dx} + \frac{1}{2}\sigma^2 x^2 \frac{d^2U}{dx^2} - rU = 0 \quad \forall x \in [0, X^*). \tag{7}$$

It is easy to verify that the expression

$$U(x) = c_1 x^{p_1} + c_2 x^{p_2}$$

satisfies the ODE in equation (7). Here p_1 and p_2 are roots of the quadratic equation

$$\frac{1}{2}\sigma^2 p^2 + (\mu - \frac{1}{2}\sigma^2)p - r = 0, \tag{8}$$

and c_1, c_2 depend on the boundary condition. We recall that r is positive. This ensures that the quadratic equation (8), has exactly one positive root. The term in the utility function with the non-positive root disappears in order to prevent the utility function from exploding at the origin. We denote the only positive root of quadratic equation (8) by p^*. Thus the utility function has the form $U(x) = cx^{p^*}$ for some constant c.

At boundary X^*, the utility from continuing equals the reward from stopping,

$$U(X^*) = 1 - e^{-\alpha(X^* - K)} = c(X^*)^{p^*},$$

so, we have

$$c = \frac{U(X^*)}{(X^*)^{p^*}},$$

and

$$U(x) = U(X^*)\left(\frac{x}{X^*}\right)^{p^*} = \left(1 - e^{-\alpha(X^* - K)}\right)\left(\frac{x}{X^*}\right)^{p^*} \quad \forall x \in [0, X^*).$$

Under utility-indifference pricing, utility $U(x)$ has the equivalent asset value $V(x)$ as the following equation,

$$U(x) = 1 - e^{-\alpha V(x)} = \left(1 - e^{-\alpha(X^* - K)}\right)\left(\frac{x}{X^*}\right)^{p^*}. \tag{9}$$

The last remaining step is to maximize $U(\cdot)$ over all possible values of X^* : $X^* > K$. This identifies the optimal exercise barrier X^*. By the first order condition of this maximization problem, X^* has to be the unique root of the following equation,

$$X^* = K + \frac{1}{\alpha}\log\left(1 + \frac{\alpha}{p^*}X^*\right). \tag{10}$$

With the above assumptions, we derive explicit formulas to obtain optimal exercise thresholds and evaluate the value of an R&D intangible asset. The next section considers another dimension of the R&D technology investment--the industry competition effects.

2.4 Industry Competition Effects

This section evaluates the option to invest in a new market that is shared among many competitors. The firm has to decide when to hold its option and wait, and when to enter the market. All of this must take into account for the possible loss of early movers advantage to becoming later movers in the market. For simplicity, with each competitor entering the market the project value of the firm is reduced by an uncertain percentage, modeled as a negative exponential jump. Therefore the impact of competition is stochastic. Another randomness to the firm is the timing of when competitors choose to exercise their investment option. Such a situation is conveniently modeled by a Poisson process which takes account of the random points in time when the gross project value V drops. The question that arises now is what optimal strategy the firm should pursue when it faces these competitive entries. Different from what is usually argued in literature, we find that a preemption strategy is not always feasible in the case of exogenous stochastic competition.

Suppose that R&D intangible asset value process distributes as e^{X_t}, where X_t is given by

$$X_t = x + \mu t + \sigma B_t - \sum_{k=1}^{N_t} Y_k \ ; \ t \geq 0,$$

where B_t is the general evolution of the asset process (a Brownian motion), N_t is the competition intensity (a Poisson process with intensity λ), and Y_t is the competition strength faced by the firm (a exponential distributed with parameter α) We face the same problem: find a real function V and a stopping rule τ^*, such that $V(x) = \sup_\tau E^x(g(X_\tau)) = E^x(g(X_{\tau^*}))$.

First, the infinitesimal generator of the process X (refer to Mordecki (1999)) is

$$(\mathrm{A}^X f)(x) = \frac{1}{2}\sigma^2 f''(x) - \mu f'(x) + \lambda\int_0^\infty (f(x-y) - f(x))dF(y). \tag{11}$$

For $X < X^*$, it is easy to see that $V(x) = (e^{X^* - K})e^{p(x - X^*)}$ is a solution to the following Bellman equation in the continuation region.

$$-rV + \mathrm{A}^X V = 0. \tag{12}$$

Substitute $(e^{X^*-K})e^{p(x-X^*)}$ to (11), we have

$$(\mathrm{A}^X V)(x) = (e^{X^*-K})e^{p(x-X^*)}\left(\frac{\sigma^2 p^2}{2} + \mu p + \lambda\int_0^\infty (e^{-py}-1)dF(y)\right).$$

If p^* is the only positive solution to the following equation and $p^* > 1$,

$$\frac{\sigma^2 p^2}{2} + \mu p + \lambda\int_0^\infty (e^{-py}-1)dF(y) = r, \tag{13}$$

this is equivalent to

$$\frac{\alpha^2}{2}p^3 + (\mu + \frac{\sigma^2}{2}\gamma)p^2 + (\mu\gamma - \lambda - r)p - \gamma^2 = 0, \tag{14}$$

then this solution satisfies the Bellman equation (12), so

$$V(x) = (e^{X^*} - K)e^{p(x-X^*)}$$

is indeed the solution. For the optimal investment threshold, our another condition is

$$\left.\frac{\partial V(X)}{\partial X}\right|_{X=X^*} = 0,$$

which leads to the following solution for X^*

$$X^* = \log\left(K\frac{p^*}{p^*-1}\right). \tag{15}$$

Then, the solution to the optimal stopping problem is

$$\tau^* = \inf\{t \ge 0\ ;\ X_t \ge X^*\}$$

$$V(x) = \begin{cases} (e^{X^*} - K)\exp\{p^*(x - X^*)\} & if\ x \le X^*, \\ e^x - K & if\ x > X^*. \end{cases}$$

3. INFORMATION REVELATION AND LEARNING EFFECTS

In real situations the value process of an R&D intangible asset is unobservable. Before investing, the true value can only be inferred from noisy market information or market research. We are forced to use some sort of estimation technique to make decisions. The best strategy to solve this problem is to employ an optimal filtering technique that can progressively learn noisy asset information. Under incomplete asset information (or unobserved system), optimal filtering is the best solution.

Investors with incomplete information face uncertainties about asset processes. In order to begin with a formal analysis, we will first describe the continuous-time stochastic processes governing the true R&D asset value-noise and the observed asset value. Assuming that the true asset value follows a lognormal stochastic process, specified as follows:

$$dX_t = \mu_X X_t dt + \sigma_X X_t dB_X. \tag{16}$$

The true value is obscured by noise, meaning that its value is estimated with error. We model noise also as a lognormal process that reverts toward its long-run, neutral value of one, with dynamics as follows:

$$dY_t = (-\kappa \ln Y_t)Y_t dt + \sigma_Y Y_t dB_Y. \tag{17}$$

In a lognormal setting, the observed value is defined as the product of the true asset value and noise, i.e., $Z_t = X_t Y_t$. Using this relation, the stochastic process that describes the observed value is

$$dZ_t = \left(\mu_X - \kappa\ln\left(\frac{Z_t}{X_t}\right)\right)Z_t dt + \sigma_X Z_t dB_X + \sigma_Y Z_t dB_Y.$$

The correlation between instantaneous changes in the observed value and the true value is

$$\rho \equiv corr(dX_t, dZ_t) = \frac{\sigma_X}{\sqrt{\sigma_X^2 + \sigma_Y^2}},$$

which increases as noise volatility σ_Y diminishes. We can think of ρ as the instantaneous information ratio. The time-filtered asset value $\hat{X}(t) = E[X_t \mid \Phi_t]$ provides an efficient unbiased estimate of the true asset value that is conditional on all available information through time t. Its dynamics are determined by the optimal filtering techniques of Lipster and Shiryayev (2001), which produce a stochastic differential equation for $\hat{X}_t$:

$$d\hat{X}_t = \mu_X \hat{X}_t dt + S_t \hat{X}_t \sigma_Z dB_Z, \tag{18}$$

where S_t is the error variance, which is

$$S_t = \rho^2 \left(1 + \frac{\kappa \gamma_t}{\sigma_X^2}\right), \tag{19}$$

and where

$$\frac{d\gamma(t)}{dt} = \sigma_X^2 - \frac{\rho^2 (\sigma_X^2 + \gamma(t)\kappa)^2}{\sigma_X^2}. \tag{20}$$

When γ reaches a steady state, new information can not improve the estimation. We have

$$\gamma_{ss} = \frac{\sigma_X^2 (1-\rho)}{\rho \kappa} \tag{21}$$

and

$$S_{ss} = \rho^2 \left(1 + \frac{\kappa \gamma_{ss}}{\sigma_X^2}\right). \tag{22}$$

Assuming error variance S_t reaches a steady state quickly and applying the results from section 2, we get the characteristic operator of $\hat{X}$ to be

$$\mathrm{A}^{\hat{X}} \phi = \mu_X x \frac{\partial \phi}{\partial x} + \frac{1}{2} S_{ss}^2 \sigma_Z^2 x^2 \frac{\partial^2 \phi}{\partial x^2}. \tag{23}$$

Under the learning mechanism, the optimal solution V must satisfy the following Bellman ODE in the continuation region

$$-rV + \mathrm{A}^{\hat{X}} V = 0,$$

that is

$$-rV + \mu_X x \frac{\partial V}{\partial x} + \frac{1}{2} S_{ss}^2 \sigma_Z^2 x^2 \frac{\partial^2 V}{\partial x^2} = 0. \tag{24}$$

It's easy to see that the solution is x^P, where P satisfies the following equation:

$$-r + \mu_X P + \frac{1}{2} S_{ss}^2 \sigma_Z^2 P(1-P) = 0; \tag{25}$$

at the same time, these solutions must satisfy the following boundary conditions

$$V(0) = 0$$
$$V(X^*) = X^* - K$$
$$V'(X^*) = 1,$$

where $V'(X^*) = 1$ is a smooth pasting condition. Based on these explicit solutions, it's easy to make numerical computations to discover some comparative statics. Through simple calculation, the solution of P is

$$P = \frac{1}{2 S_{ss}^2 \sigma_Z^2} \left(-2\mu_X + S_{ss}^2 \sigma_Z^2 + \sqrt{(2\mu_X - S_{ss}^2 \sigma_Z^2)^2 + 8 S_{ss}^2 \sigma_Z^2 r} \right), \tag{26}$$

the value function $V(x)$ is

$$V(x)=\begin{cases}\dfrac{(P-1)^{P-1}}{P^P K^{P-1}}x^P & if\ x\le X^*,\\ x-K & if\ x>X^*.\end{cases} \quad (27)$$

4. NUMERICAL RESULTS

Figures 1 through 4 consider risk sensitive effects in optimal R&D valuation and investment thresholds. In Figures 1 and 2, we set $K=2,\mu=2\%<r=4\%$, In Figures 3 and 4, we set $K=2,\mu=8\%>r=4\%$, they represent two different situations and will be explained below. Figures 1 and 3 graph equivalent R&D project value V under different volatilities σ and various current asset value x. Figures 2 and 4 plot investment thresholds X^* under different risk aversion parameter α and various volatilities σ. They imply that the asset value process parameters μ and σ as well as the executive impatience parameter r exert their impact through the coefficient p^*. As shown in Figures 1 through 4, the coefficient p^* completely captures the dependency of the executive's optimal exercise barrier X^* as well as of the R&D value V on these problem parameters.

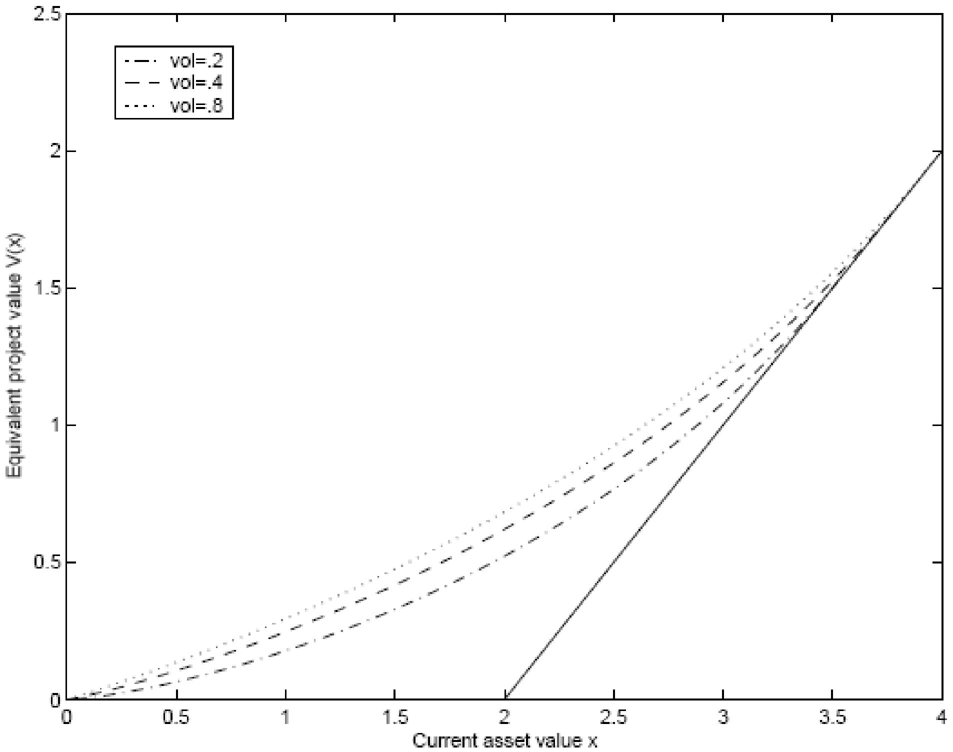

Figure 1: **Equivalent project value under risk sensitive effects I**. Model parameters of this figure are $K=2,\mu=2\%<r=4\%,\alpha=0.6$, and $\sigma=vol=0.2,0.4,0.8$.

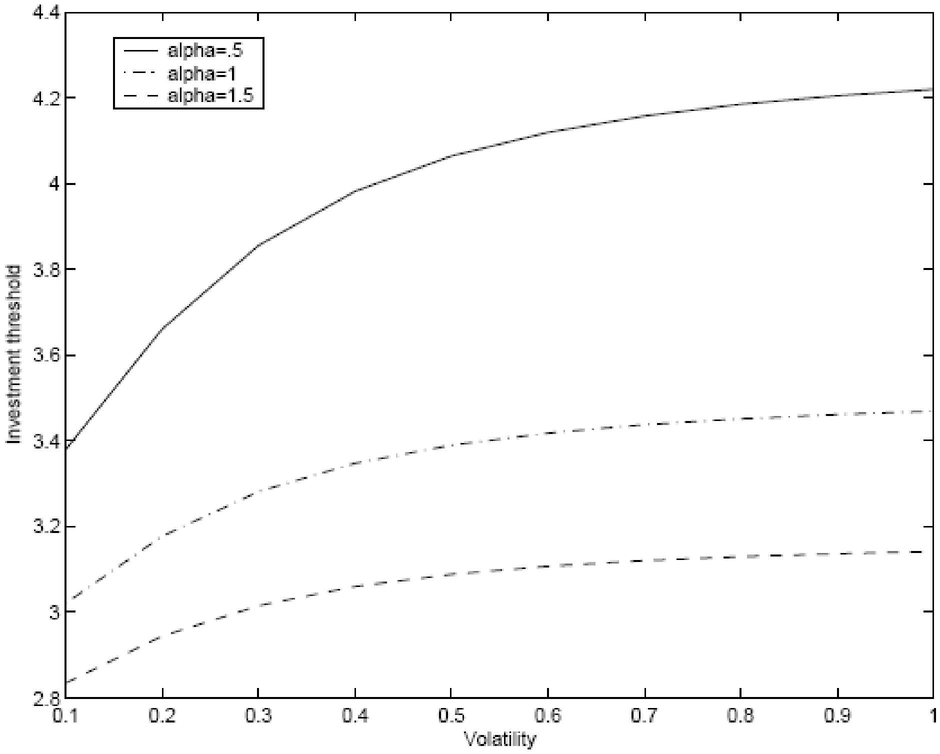

Figure 2: **Investment threshold under risk sensitive effects I**. Model parameters of this figure are $K = 2, \mu = 2\% < r = 4\%$, and $\alpha = alpha = 0.5,1,1.5$.

The variation in the R&D value, as the volatility parameter changes are shown in Figures 1 and 3. It depends on the relative magnitude of μ and r. When $\mu < r$, the executive perceives the rate of price appreciation to be too slow and hence favors an increase in volatility. This in turn will increase the probability of hitting the exercise barrier within a reasonable (limited) holding period. These effects are shown in Figure 1. R&D project value increases with volatility increases. On the other hand, when $\mu > r$, the executive perceives the rate of price appreciation to be fast enough; so there is a high probability of hitting the exercise barrier within a reasonable (limited) holding period. Hence, she is averse to disturbances because volatility exists in the process. These effects are shown in Figure 3; R&D project value decreases with volatility increases.

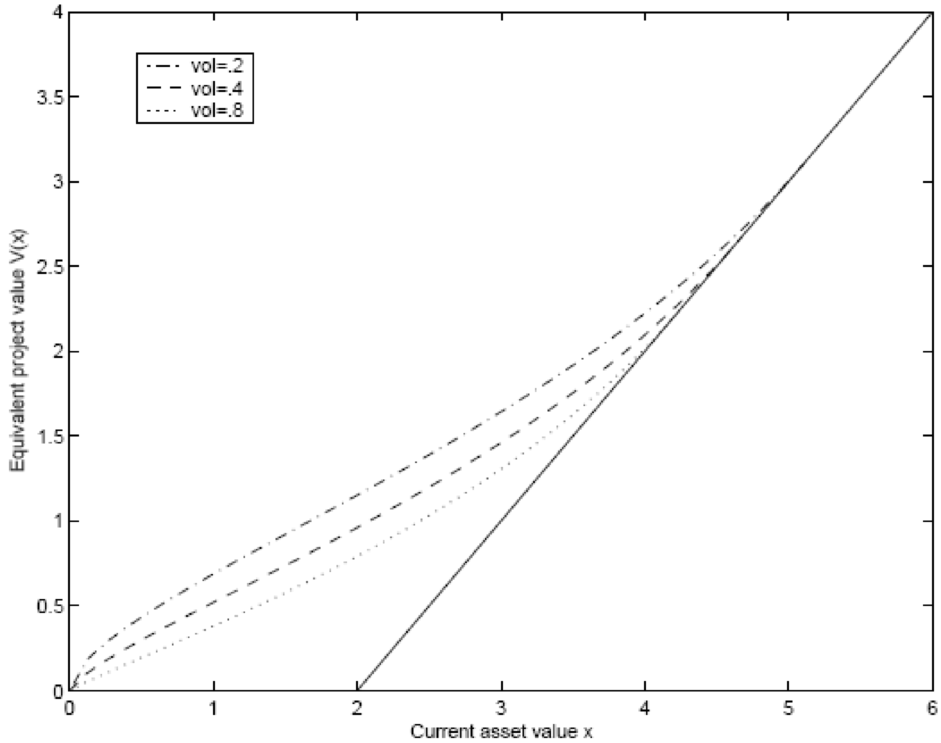

Figure 3: **Equivalent project value under risk sensitive effects II**. Model parameters of this figure are $K = 2, \mu = 8\% > r = 4\%, \alpha = 0.6$, and $\sigma = vol = 0.2, 0.4, 0.8$.

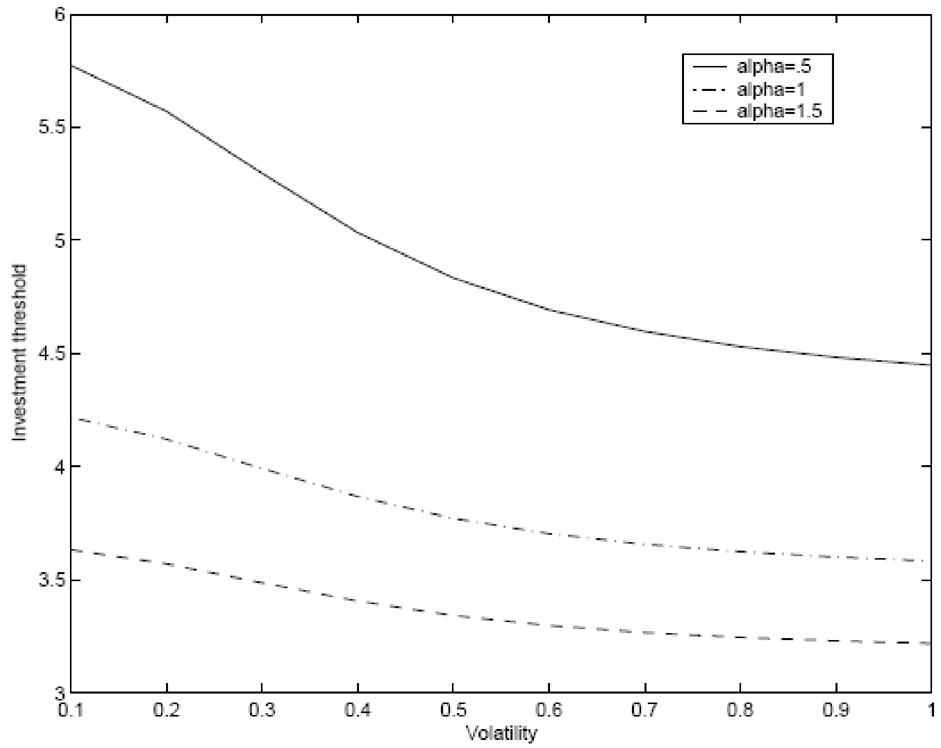

Figure 4: **Investment threshold under risk sensitive effects II**. Model parameters of this figure are $K = 2, \mu = 8\% > r = 4\%$, and $\alpha = alpha = 0.5, 1, 1.5$.

From another dimension, the variations in the R&D investment threshold, as the volatility parameter changes are shown in Figures 2 and 4. The variations also depend on the relative magnitude of μ and r. As we know, due to boundary conditions, the critical value X^* is the tangent point of $V(X)$ and the line $X-K$. In normal circumstances, where $\mu < r$, and we apply the results from Figure 1 to our equation, the investment thresholds increase with volatility increases. Similarly, in the case, where $\mu > r$, the investment thresholds decrease with volatility increases after we apply the results from Figure 3.

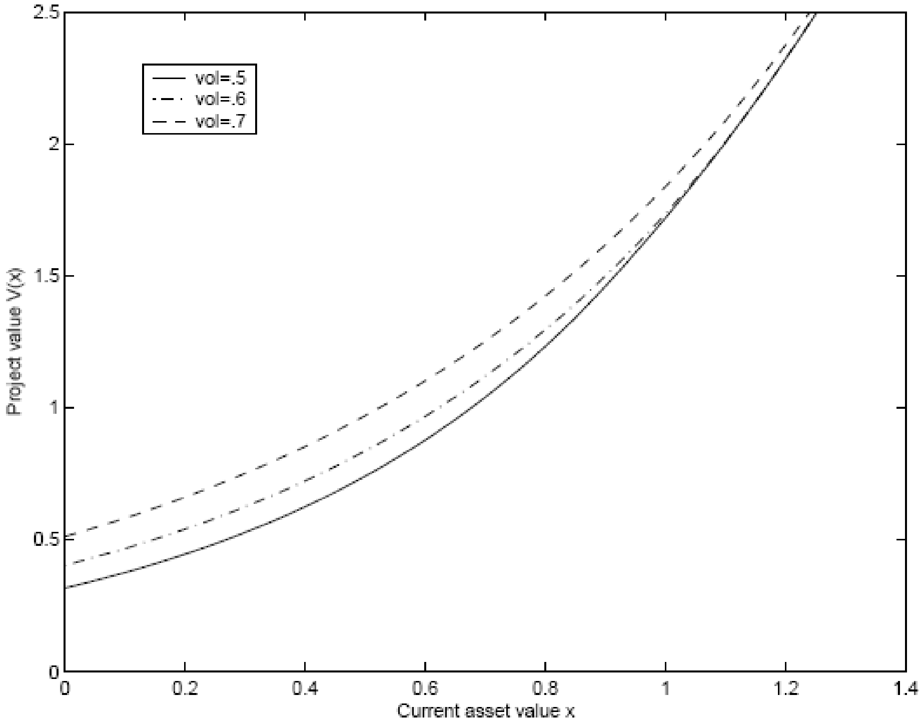

Figure 5: **R&D project value under industry competition**. Model parameters of this figure are $K=1, r=4\%, \mu=0.1, \lambda=0.5, \gamma=0.5$, and $\sigma = vol = 0.5, 0.6, 0.7$.

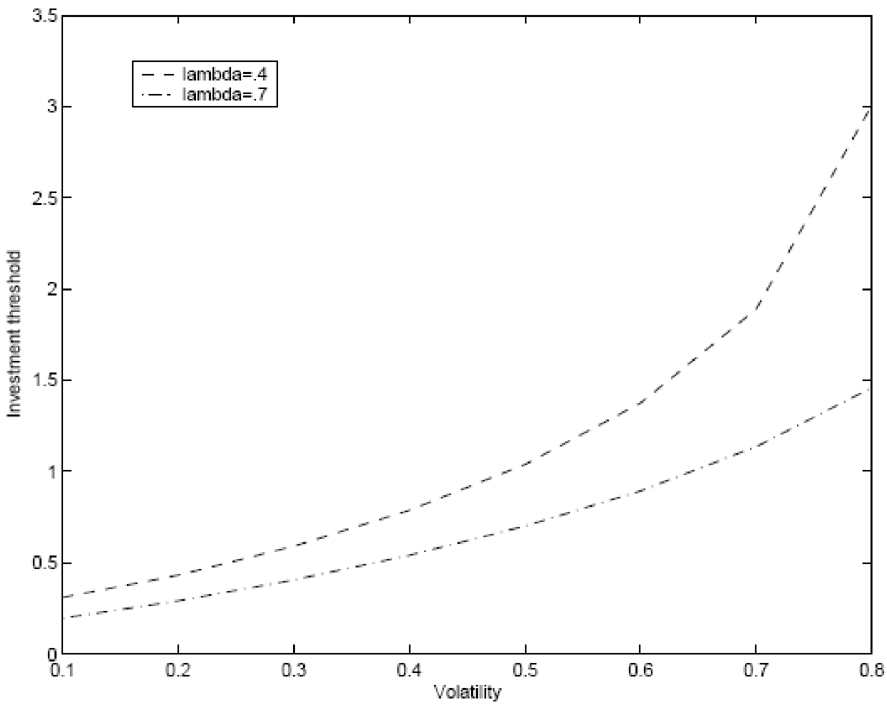

Figure 6: **Investment threshold value under industry competition I**. Model parameters of this figure are $K=1, r=4\%, \mu=0.1, \gamma=0.5$, and $\lambda = lambda = 0.4, 0.7$.

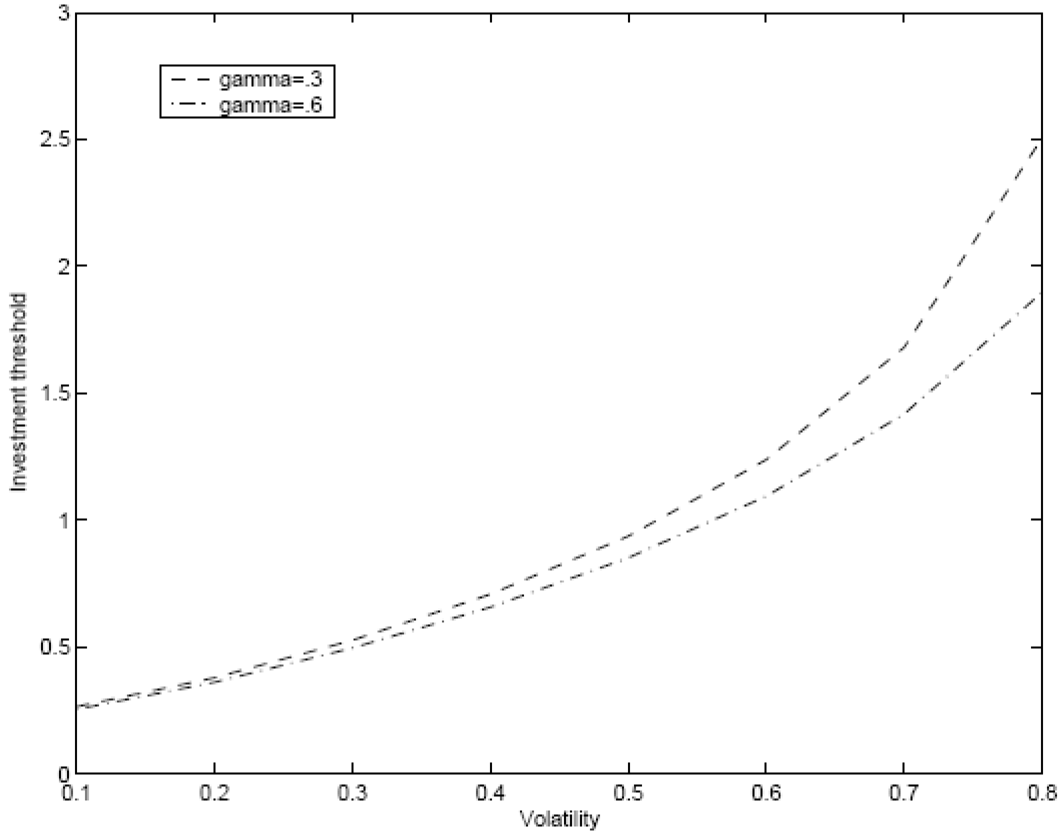

Figure 7: **Investment threshold value under industry competition II**. Model parameters of this figure are $K = 1, r = 4\%, \mu = 0.1, \lambda = 0.5$, and $\gamma = gamma = 0.3, 0.6$.

Figures 5 through 7 consider industry competition in R&D project valuation. In all these figures, we set $K = 1, r = 4\%, \mu = 0.1$. Figure 5 plots R&D project value V under different volatilities σ. From this figure, we know that $V(x)$ increases when σ increases, and so does the critical value X^*. This is because uncertainty will increase the value of an R&D project. As a result, when the market or economic environment becomes more uncertain, the R&D project value will rise. Figure 6 plots the impact of changing competition arrival rate λ on investment thresholds X^*. The critical X^* decreases with λ increases. This is intuitive since higher λ means a lot of competitors are waiting to enter the market. We have to pay a higher market risk premium when the investment is delayed, so this leads to earlier investment when λ is large. Figure 7 plots the impact of changing competition strength γ on investment thresholds X^*. The critical X^* decreases with γ increases. The intuition behind the study is the same as above. Higher γ means the competition is severe in the market, we also have to pay higher market risk premium when the investment is delayed, so this also leads to earlier investment when γ is large.

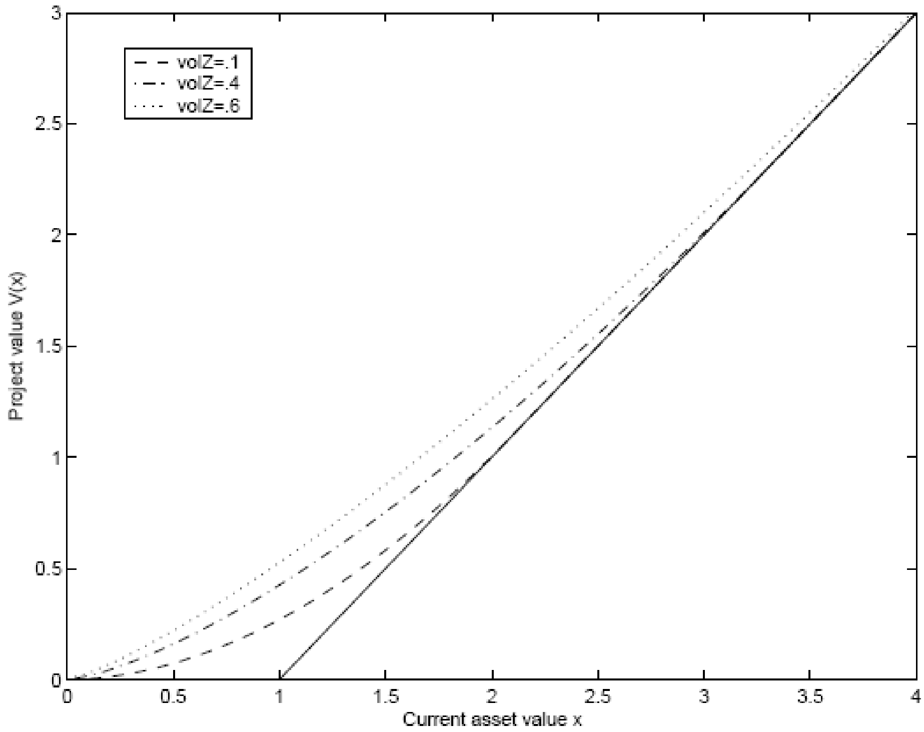

Figure 8: **R&D project value under learning effects**. Model parameters of this figure are $K=1, r=4\%, \mu_X=2\%, \sigma_X=0.2, \rho=0.2$, and $\sigma_Z=volZ=0.1,0.4,0.6$.

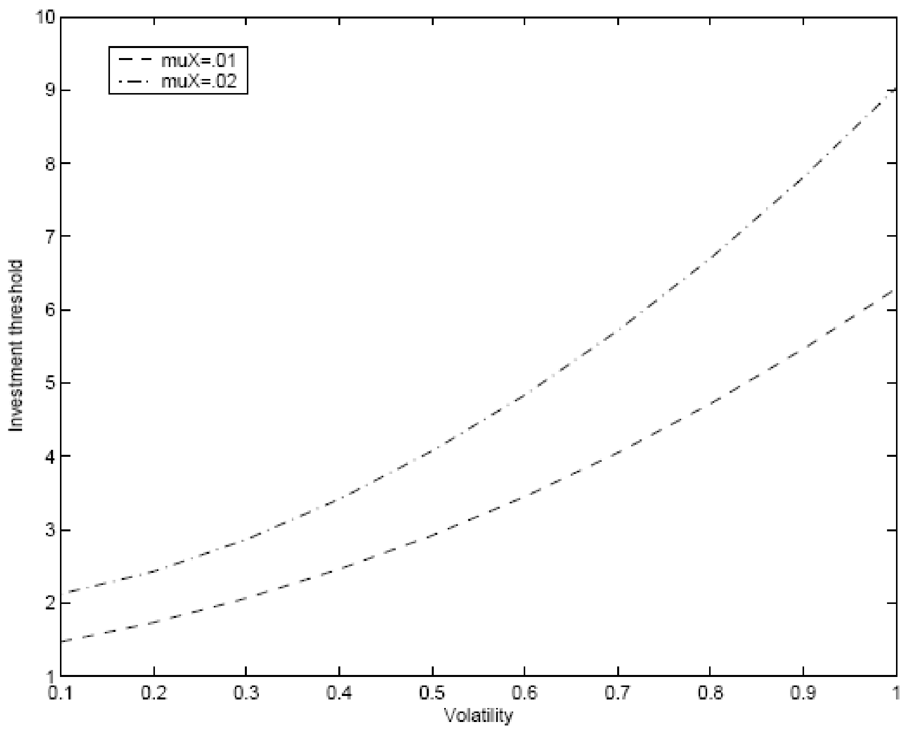

Figure 9: **Investment threshold under learning effects**. Model parameters of this figure are $K=1, r=4\%, \sigma_Z=0.4, \sigma_X=0.2, \rho=0.5, \kappa=0.7$, and $\mu_X=muX=1\%,2\%$.

Figures 8 and 9 consider learning effects on R&D project valuation. In all these figures, we set $K=1, r=4\%, \kappa=0.7$. Figure 8 graphs R&D project value V under different volatilities σ_Z. From this figure we know that $V(X)$ increases when σ_Z increases, and so does the critical value X^*. This is also because uncertainty will increase the value of a R&D project. As a result, when the market or economic environment becomes more uncertain, the value of the R&D project will increase. Figure 9 plots the impact of changing growth rate μ_X on investment thresholds. The critical X^* increases with μ_X increases. This is also due to boundary conditions. Because critical value X^* is the tangent point of $V(X)$ and $X-K$; higher μ_X means higher option value $V(X)$, so higher tangent point.

5. CONCLUSION

By considering the industry competition and incomplete information in R&D investments, this study provides an optimal valuation and deeper understanding of the influences of technological progress and information revelation on investment decisions.

First part of this study determined the optimal valuation and exercise policy of an R&D project from utility-based pricing. From the risk attitude of a manger, we presented some findings not found in previous literatures. For instance, the R&D value increases with the volatility of the R&D asset price process if $\mu < r$, but decreases with volatility when $\mu > r$). The influence of industry competition on manager's decision-making is also considered. From numerical analysis, the critical exercise point X^* decreases with λ increases and also γ increases. The intuition is that higher λ means that many competitors are waiting to enter the market, and higher γ means that competition is severe in the marketplace. In these two cases, a manager have to pay higher market risk premium when the investment is delayed.

Second part of this study considered the information revelation and learning effects of an R&D project. Applying optimal filtering theory, the best estimation based on market research signals is obtained, which is the best solution based upon incomplete market information.

The transparent R&D valuation formula of this study enables companies to calculate the expense of R&D investments more easily. A natural extension of our work is to embed our model in a principal agent setting to examine the alignment of the relative benefits between managers and principals, and how to reward managers for their performance.

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