BELIEFS-UPDATING AND DECISION-MAKING WITH INFORMATION REVELATION AND SIGNALING

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ABSTARCT

The axioms of the expected-utility maximization theorem assure the existence of a utility function and a conditional-probability function such that choice among a set of lotteries under consideration would be equivalent to choice made by referring to the expected utilities of the prizes associated with the alternative lotteries. Whereas the expected-utility maximization theorem provides a theoretical foundation for analysis of decision-making under uncertainty, how to represent the beliefs and beliefs-updating associated with the choices of decision-makers within an interactive context appears to be critical in theoretical or empirical research on service-management and other managerial topics. This paper discusses beliefs-updating and decision-making with information revelation and signaling by reviewing some of the influential solution concepts and approaches developed to deal with the related issues in such a consistent and illuminating perspective as would facilitate research and instruction of this concern.

Keywords: information; signaling; beliefs; choice; decision making

1. INTRODUCTION

Humans acquire information with regard to themselves and the world outside themselves and interpret information in various ways. Crotty (1998) proposed the hypothetical example of Tycho Brahe and Johannes Kepler standing together on a hall at sunrise. Brathe thought that the sun circles the earth, while Kepler believed that the earth circles the sun. As asked by Crotty with regard to the perceptions of Brahe and Kepler about the natural phenomenon and their interpretations, a question may be raised about what they see as they watch the sun appear at daybreak, or more precisely about whether Brathe see the sun move above the earth’s horizon, while Kepler sees the horizon dip below the sun. Crotty asserted that a chair, as a chair, is constructed, sustained and reproduced through social life, while it may exist as a phenomenal object regardless of whether any consciousness is aware of its existence. The diversification in human information acquisition and interpretation suggests the difficulty in understanding human beliefs-updating and decision-making and is investigated in one line of research in economics and management science by considering beliefs-updating within an interactive context with information revelation and signaling. In this paper, some of the influential solution concepts and approaches developed to deal with the problem of beliefs-updating and decision-making within an interactive social context will be discussed in a consistent and illuminating perspective to facilitate theoretical or empirical research on service-management and other managerial topics and instruction of a similar concern.

2. CHOICE UNDER UNCERTAINTY

Within an interactive context, decision-makers are generally assumed to be rational and intelligent (Myerson, 1991). As a rational and intelligent decision-maker makes decisions consistently in pursuit of her own objectives and uses information available to make inferences about the situation, consistency in decision making appears to be an important issue to be addressed in managerial and economic analysis. One axiomatic approach to this issue is based on acceptable beliefs regarding the preferences of a decision-maker. As mentioned by Sen (1996) with regard to an individual's preferences and act of choice, an individual's act of choice would be characterized by such specific features as the identity of the chooser, the set of items or the menu over which choice is being made, and the set of social relations and norms which may influence choice of social actions. Furthermore, he asserted that the preferences of an individual over a set of comprehensive outcomes should be distinguished from the conditional preferences over culmination outcomes given the acts of choice. A decision-maker's choice under uncertainty would be associated considerably with the decision-maker's conditional preferences over observable and unobservable culmination outcomes and may be illuminated by the derivation of the expected-utility maximization theorem and its applications in various research fields.

As Myerson (1991) pointed out, a rational decision-maker's preferences may be expected to satisfy some basic properties which can be presented as a list of axioms. Starting with decision-making as choice among lotteries in describing choice under uncertainty, a lottery is defined to be any function f which specifies a nonnegative real number $f(x/t)$ for every x in X and every t in Ω, where X denotes the set of possible prizes which the decision-maker could ultimately get and Ω denotes the set of possible states within a probability measure space of the triple $\{\Omega, F, P\}$. Under the axioms of interest, completeness, and transitivity, the preferences of a decision-maker are believed to be such that he is never indifferent between all the prizes in the set of possible prizes and his preferences should always form a complete transitive order over the set of alternatives. Axioms of monotonicity and continuity assert that an alternative with a higher probability of getting a better lottery among a set of lotteries is always better than one with a lower probability of getting the same better lottery among the same set of lotteries and a lottery that is ranked between two lotteries just is as good as some randomization between these two. Furthermore, by the axiom of relevance only the possible states are relevant to the decision-maker. Within the context of a probability measure space $\{\Omega, F, P\}$, a decision-maker takes into consideration only relevant events S in $\Xi = \{S | S \in F \wedge S \neq \phi\}$ such that $P(S)$ is not equal to zero. With mutually exclusive events, one of which would occur, if a decision-maker must choose between two alternatives and he would prefer the first alternative in each event, the substitution axioms assert that he must prefer the first alternative to the other one even before he learns which one of these mutually exclusive events would occur. As Myerson stated with regard to the expected-utility maximization theorem, with the establishment of these axioms, choice among a set of lotteries L would be equivalent to choice made by looking at the expected utilities of the prizes associated with the lotteries in the set L. That is, $\forall f, g \in L$, $\forall S \in \Xi$, $f \geq_S g$ if and only if $E_p(u(f)|S) \geq E_p(u(g)|S)$, where the axioms assure the existence of a utility function $u : X \times \Omega \to R$ and a conditional-probability function p such that $p : \Xi \to \Delta(\Omega)$ with $\Delta(\Omega) = \left\{ q : \Omega \to R \middle| \sum_{\omega \in \Omega} q(\omega) = 1 \wedge q(\omega) \geq 0, \forall \omega \in \Omega \right\}$.

Within a managerial context with consumption and provision of commodities traded in markets, the types of prizes as contained in sets of possible prizes available to decision-makers of various types would include, but not limited to, quantities of goods to be consumed, monetary returns and wealth, revenues, profits, and in some managerial contexts such non-physical or non-monetary objects as probabilities of on-time delivery of commodities or services to customers. As the utility of a lottery is the expected utility of its prizes, the properties of the expected utility function and the prizes of lotteries play crucial parts in research dealing with uncertainty. However, in analysis with a focus on elements excluding preferences, risk-neutrality for the preferences of a decision-maker is generally assumed in analysis. As an affine transformation of an expected utility function would preserve the expected utility property which describes a decision-maker's choice behavior and is also an expected utility function, with the assumption of risk neutrality for the preferences of a decision-maker, there would exist an affine transformation of the expected utility function of a decision-maker such that the expected values of the contingent prizes associated with an alternative would be the expected utility of the contingent prizes with the alternative . However, in a cost model, the type of the prize would be the cost associated with the operations under a decision-maker's choice. In this case under the assumption of risk neutrality, it would be minimization of the expected cost or maximization of the negative of the expected cost which would be relevant in decision-making.

3. OUTCOME SPACE AND HISTORY

Whereas the axioms of the expected-utility maximization theorem assure the existence of a utility function and a conditional-probability function such that choice among a set of lotteries L would be equivalent to choice made by referring to the expected utilities of the prizes associated with the lotteries in the set L, how to represent the outcome spaces and the conditional-probability function associated with the subjective choice of a decision-maker appears to be one of the critical issues to be considered in theoretical or empirical research. One line of research on decision-making appears to understand a decision-maker's beliefs and beliefs-updating with respect to the culmination of outcomes within a measure-theoretic information structure by examining outcome spaces, information partitions or probability measures as perceived or possessed by decision-makers within various sequential interactive contexts. Billingsley (1986) described partial information as a subclass $\tilde{A}$ of F within a probability measure space of the triple $\{\Omega, F, P\}$, where Ω is a nonempty outcome space and P is a probability measure defined on F. With regard to the perception of an observer with the information in $\sigma(\tilde{A})$, he is meant to know not the point ω drawn, but the equivalence class containing it. As Fudenburg and Tirole (1991) stated with regard to the concept of a correlating device represented as a triple $\{\Omega, \{H_i\}, P\}$, the information partition $\{H_i\}$ possessed by a decision-maker is a partition of a finite state space Ω with an $h_i(\omega)$ assigned to each ω, where $h_i(\omega)$ consists of those states that a decision-maker regards as possible with the true state ω. A decision-maker has no information at all beyond her prior if her partition is the one-element set Ω, while he is well informed and has full information if each $h_i(\omega)$ in her partition is a singleton, containing exactly one element ω in the set Ω. In the latter case, each culmination outcome is observed and distinguished by the decision-maker.

In order to understand a decision-maker's beliefs and beliefs-updating in an interactive managerial context, a general outcome space can be viewed as a product outcome space formed by the Cartesian product of the nature, demand, and supply outcome spaces (Chen, 2011). For an interactive context with agents of various types, a general product outcome space confronting all the agents as the nature, consumers, providers, and governments may be represented by the agents' types and strategies as the Cartesian product of $\Omega = \Theta \times A$, where $\Theta = \{(\theta_1, ..., \theta_I) | \theta_i \in \Theta_i, i = 1, ..., I\}$ and $A = \{(s_1, ..., s_I) | s_i \in S_i, i = 1, ..., I\}$ with a finite number of types θ_i in Θ_i and feasible strategies s_i in S_i for each agent i. The product outcome space may be so general that it would be time invariant within the time span under consideration in that the possible outcomes pertinent to each point of time are included therein and $\Omega = \Omega^1 = ... = \Omega^T$. This would correspond to the assertion of Sen (1996) that a set of comprehensive outcomes should include the entire choice process. As the evolving of time indexed as stages or periods, the actions taken by all the agents would be the strategies they put into effect at various distinct stages or periods and would form a history completely or partially perceivable to all the agents in interaction. All the agents may update their beliefs with respect to this interactive context by including the completely or partially perceivable history into her information pool and make inferences accordingly to form posterior beliefs about this context, represented symbolically by the measure space $\{\Omega, F_\Omega, p_\Omega\}$. Within a multi-stage interaction context, the outcome space may be extended to $\tilde{\Omega} = \Omega^1 \times ... \times \Omega^T$, where $\Omega^t = \{((\theta_1^t, ..., \theta_I^t), (s_1^t, ..., s_I^t)) | \theta_i^t \in \Theta_i \wedge s_i^t \in S_i, i = 1, ..., I\}, t = 1, ..., T$; that is, the extended outcome space would be a set of time/stage-indexed combinations of type profiles and strategy profiles. Similarly, each agent may update her beliefs with respect to this multi-stage interactive context by including completely or partially perceivable history into her information pool and make inferences accordingly to update her posterior beliefs regarding this context, represented symbolically by the measure space $\{\tilde{\Omega}, F_{\tilde{\Omega}}, p_{\tilde{\Omega}}\}$. In the case that only one agent takes an action at each stage, the other agents are meant to take the action of "making no decision and doing nothing" at each stage. Let a history space $\tilde{\Omega}^t$ refer to $\Omega^1 \times ... \times \Omega^t, t \geq 1$. Thus a history $\tilde{\omega}^t \in \tilde{\Omega}^t$ at the end of stage t would be a time-indexed type-action sequence taking the form of $(((\theta_1^1,...,\theta_I^1),(s_1^1,...,s_I^1)),...,((\theta_1^t,...,\theta_I^t),(s_1^t,...,s_I^t)))$ and would become the history at the beginning of stage $t+1$. With time-invariant types, a history at the end of stage t would be the type-action sequence $((\theta_1,...,\theta_I),(s_1^1,...,s_I^1)),...,((\theta_1,...,\theta_I),(s_1^t,...,s_I^t))$ and may be viewed as an action path conditional on the time-invariant type profile of $(\theta_1,...,\theta_I)$ at the end of stage t.

4. BELIEFS AND CHOICE WITH COMPLETE INFORMATION

How an agent chooses a strategy at a distinct point of time depends considerably on the beliefs he possesses at that point of time. With the evolving of time and history, the information possessed by decision-maker i regarding $\tilde{\Omega}^t$ may be represented by the information partition $\{H_i^t\}$ structured in such a way that a culmination history as he perceives at the end of stage t would take the following form: $(h_i^1((\theta_1^1,...,\theta_I^1),(s_1^1,...,s_I^1)),...,h_i^t((\theta_1^t,...,\theta_I^t),(s_1^t,...,s_I^t)))$, where the partial information set of $h_i^k((\theta_1^k,...,\theta_I^k),(s_1^k,...,s_I^k))$ contains the culmination outcome $((\theta_1^k,...,\theta_I^k),(s_1^k,...,s_I^k))$ and sample points equivalent to $((\theta_1^k,...,\theta_I^k),(s_1^k,...,s_I^k))$ in Ω^k with $1\le k\le t$. With complete information, the culmination history is completely perceived by each agent and the partial information set of $h_i^t((\theta_1^t,...,\theta_I^t),(s_1^t,...,s_I^t))$ would contain merely the combination of the culmination type profile and action profile at the end of stage t, that is, $((\theta_1^t,...,\theta_I^t),(s_1^t,...,s_I^t))$. With complete information, the culmination history at the end of stage t and thus the beginning of stage $t+1$ would be the type-action sequence: $\tilde{\omega}^t =$ $(((\theta_1^1,...,\theta_I^1),(s_1^1,...,s_I^1)),...,((\theta_1^t,...,\theta_I^t),(s_1^t,...,s_I^t)))$. The payoff profile pertinent to a specific history at the terminal stage T can be represented by $u(\tilde{\omega}^T)$, where $u(\tilde{\omega}^T)=(u_1(\tilde{\omega}^T),...,u_I(\tilde{\omega}^T))$. For each agent, her payoff at the termination stage would depend not merely on the types and strategies of her own, but also those of the other agents at each point of time within this interactive context. Agents' beliefs regarding the culmination history at the end of each stage would play a crucial role in the sequential action choice taken by the agents.

In the presence of time-invariant types and complete information, the culmination history as perceived at the end of stage t would thus be the type-action sequence: $\tilde{\omega}^t(\theta)=$ $((\theta,(s_1^1,...,s_I^1)),...,(\theta,(s_1^t,...,s_I^t)))$, where $\theta=(\theta_1,...,\theta_I)$. Let (ϕ) denote the action of "making no decision and doing nothing". Within an interactive context with only one agent taking an action at each stage and agents being indexed in such a way that agent i takes her move at the *ith* stage, in the case of time-invariant types with complete information, the culmination history at the end of stage t with $1\le t<I$ would be the type-action sequence of $((\theta,(s_1^1,...,s_I^1)),...,(\theta,(s_1^t,...,s_I^t)))$ in which for $1\le j,k\le t$, $s_k^j\ne(\phi)$ as $k=j$ and $s_k^j=(\phi)$ as $k\ne j$, and for $t<j,k\le I$, $s_k^j=(\phi)$.

In the case of complete information with regard to the culmination history at the end of each stage, the agents would perceive the time-indexed type and action profiles. As a problem of action choice under sequential interaction is generally tackled with equilibrium solution concepts, analysis of action choice in the presence of complete information resorts considerably to the solution concept of subgame-perfect equilibrium. Let $G(h^{t-1})$ refer to the game from the beginning of stage t with history h^{t-1} as a game in its own. For a strategy profile of the whole game s with $s=((s_1^1,...,s_I^1)),...,,(s_1^T,...,s_I^T))$, a strategy profile s/h^{t-1} on $G(h^{t-1})$ is defined as the restriction of s to the histories consistent with h^{t-1} in that for each player i, s_i/h^{t-1} is the restriction of s_i to the histories consistent with h^{t-1}. As Fudenburg and Tirole (1991) mentioned with regard to action choice with observed types and actions, a strategy profile s would constitute a subgame-perfect equilibrium if, for every h^{t-1}, the restriction s/h^{t-1} to $G(h^{t-1})$ is a Nash equilibrium of $G(h^{t-1})$.

As illuminated in the definition of a subgame-perfect equilibrium, the solution concept of a subgame-perfect equilibrium depends considerably on the beliefs that equilibrium strategies would be taken by all the agents participating in the sequential interaction with complete information in that for every h^{t-1}, the restriction s/h^{t-1} to $G(h^{t-1})$ is a Nash equilibrium of $G(h^{t-1})$. With the solution concept of a subgame-perfect equilibrium with observed types and culmination actions, an equilibrium may thus be derived by backward induction and the corresponding beliefs conditional on the culmination history h^{t-1} may be examined sequentially. As applied in dynamic programming in deriving a time-indexed solution, starting from the terminal stage with the given history of h^{T-1}, at the beginning of this stage an Nash-equilibrium action profile $\tilde{a}^T$ would be a sequence of strategies as functions of the given history of h^{T-1}, that is, $\tilde{a}^T=(\tilde{s}_1^T(h^{T-1}),...,\tilde{s}_I^T(h^{T-1}))$ such that for every agent i, $\tilde{a}_i^T$ would maximize her payoff $u_i(h^{T-1},(\theta\ ,(a_i^T,\tilde{a}_{-i}^T))))$ given the culmination history h^{T-1} reveled to the agent, the observed agents' types, and the other agents' strategies $\tilde{a}_{-i}^T$ with $\tilde{a}^T\equiv(\tilde{a}_i^T,\tilde{a}_{-i}^T)$. In a model with complete information regarding types and actions, the agents' types are generally assumed to be observable and time-invariant. At the beginning of stage $T-1$, as the solution concept of a subgame-perfect equilibrium stipulates, all the agents within the interactive context would believe that the action profile to be taken at the terminal stage T would be a Nash-equilibrium action profile $\tilde{a}^T$; that is, $p_{\tilde{\Omega}}(a^T=\tilde{a}^T/h^{T-1})=1$ for every agent i if $\tilde{a}^T$ exists and is uniquely determined. Thus given the culmination history h^{T-2} and the beliefs that $p_{\tilde{\Omega}}(a^T=\tilde{a}^T/h^{T-1})=1$, the agents at the beginning of stage $T-1$ may choose among the feasible actions to form a history h^{T-1}, i.e., $(h^{T-2},(\theta,a^{T-1}))$, which would maximizes their individual payoffs. As the terminal-stage equilibrium action profile $\tilde{a}^T$ would be a sequence of strategies as functions of the given history h^{T-1}; that is, $\tilde{a}^T=(\tilde{s}_1^T(h^{T-1}),...,\tilde{s}_I^T(h^{T-1}))$, the action-choice problem confronting all the agents at the beginning of stage $T-1$ would be to choose an action profile a^{T-1} to form a history h^{T-1}, i.e., $(h^{T-2},(\theta,a^{T-1}))$ such that for every agent i, $\tilde{a}_i^{T-1}$ would maximize her payoff $u_i(h^{T-2},(\theta,(a_i^{T-1},\tilde{a}_{-i}^{T-1})),(\theta,\tilde{a}^T(h^{T-2},(\theta,(a_i^{T-1},\tilde{a}_{-i}^{T-1})))))$ given the culmination history h^{T-2} reveled to the agent, the observed agents' types, and the other agents' strategies $\tilde{a}_{-i}^{T-1}$ with $a^{T-1}\equiv(\tilde{a}_i^{T-1},\tilde{a}_{-i}^{T-1})$. The solution concept of a subgame-perfect equilibrium would thus enable the action-choice problem of the two-stage sub-game to be reduced to such a choice problem as a one-stage problem. With an optimal action profile $\tilde{a}^{T-1}$ taken at stage $T-1$, the corresponding optimal action profile $\tilde{a}^T$ taken at stage T, consistent with the history $(h^{T-2},(\theta,\tilde{a}^{T-1}))$ reveled to the agents, would be $\tilde{a}^T(h^{T-2},(\theta,\tilde{a}^{T-1}))$. In a similar way, a subgame-perfect equilibrium for a whole multi-stage game may thus be derived.

As noted by Harris (2007), in city-pair airline markets with easy entry and low customer switching costs, a new entrant is confronted with fast incumbent response. Harris analyzed a two-stage entry deterrence/accommodation game in city-pair airline markets with the aid of the solution concept of a subgame-perfect equilibrium. In their study, Ma and McGuire (1997) applied the solution concept of a subgame-perfect equilibrium to a multi-stage game in examining the action choices of the patient and the physician within an interactive medical context in which the quantity of treatment decided by the patient or the effort deicide by the physician is not contractible. However, within a medical context, the input of medical capital goods plays a critical role in the action choices of the patient, the physician, and the insurer and may be measured and verified *ex post*. To allow for substitution between a verifiable variable and an unverifiable variable at the physician's disposable and better represent the reality of incentives and behaviors in a medical context from the perspectives of the patient, the physician, and the insurer, the study of Chen, Lin and Hsieh (2008) extended the model of Ma and McGuire by including the physician's decision of the input of medical capital goods under the payment decisions made by the insurer to examine optimal medical investment and insurance under a subgame-perfect equilibrium. With positive payments to the physician, cost reduction or benefit improving is obtainable, as the study suggested, through a reduction in the social loss by restraining overinvestment in medical capital goods, while a fully prospective payment system would be a cost-minimizing payment system but not definitely a benefit-improving system.

5. BELIEFS AND CHOICE WITH INCOMPLETE INFORMATION

Within an interactive context with unobservable outcomes, agents would use all relevant information available to them to make inferences about unobservable outcomes. While unobservable outcomes within an entire outcome space $\tilde{\Omega}$ would include not only unobservable types but also actions, unobservable types are frequently encountered in decision-making and include such types as commodity quality, market demand, labor productivity, technology, knowledge, costs, and risks associated with objects or individuals relevant to decision-making. As Bayesian decision theory provides a foundation for the logical roots of a theory dealing with social interaction with beliefs-updating and decision-making such as game theory (Myerson, 1991), theoretically agents' beliefs up-dating with regard to unobserved outcomes may be understood with the aid of Bayesian decision theory and solution concepts in game theory.

As mentioned previously, the outcome space within a multi-stage interaction context may be represented symbolically by such a product space as $\tilde{\Omega} = \Omega^1 \times ... \times \Omega^T$, where $\Omega^t = \{((\theta_1^t,...,\theta_I^t),(s_1^t,...,s_I^t)) \mid \theta_i^t \in \Theta_i \wedge s_i^t \in S_i, i=1,...,I\}, t=1,...,T$. A history $\tilde{\omega}^T \in \tilde{\Omega}$ at the end of stage T would be a time-indexed type-action sequence $((\theta_1^1,...,\theta_I^1),(s_1^1,...,s_I^1)),...,((\theta_1^T,...,\theta_I^T),(s_1^T,...,s_I^T))$. To describe beliefs-updating in another perspective, the outcome space within a multi-stage interaction context may be equivalently perceived and represented by the product space $\hat{\Omega} = \Omega_\Theta^T \times .\Omega_S^T$, where $\Omega_\Theta^T = \{((\theta_1^1,...,\theta_I^1),...,(\theta_1^T,...,\theta_I^T)) \mid \theta_i^t \in \Theta_i, i=1,...,I\}$ is the time-indexed outcome space of types and $\Omega_S^T = \{((s_1^1,...,s_I^1),...,(s_1^T,...,s_I^T)) \mid s_i^t \in S_i, i=1,...,I\}$ is the time-indexed outcome space of feasible actions.

Let a history space $\hat{\Omega}^t$ refer to $\Omega_\Theta^t \times \Omega_S^t, t \geq 1$. A history $\hat{\omega}^t \in \hat{\Omega}^t$ at the end of stage t would thus be a time-indexed type-action combination of $(((\theta_1^1,...,\theta_I^1),...,(\theta_1^t,...,\theta_I^t)),((s_1^1,...,s_I^1),...,(s_1^t,...,s_I^t)))$, with the type profiles within one time-indexed sequence and the action profiles within another. Given a history of $(((\theta_1^1,...,\theta_I^1),...,(\theta_1^t,...,\theta_I^t)),((s_1^1,...,s_I^1),...,(s_1^t,...,s_I^t)))$, agents may use the information embedded in the culmination history to make inferences about the unobservable outcomes or the future events evolving within this interactive context, symbolically represented by the measure space $\{\hat{\Omega}, F_{\hat{\Omega}}, p_{\hat{\Omega}}\}$. An agent may be so poorly informed that he knows only her own types and actions within a given history. In this case, the history as agent i perceives at the end of stage t would be the set of histories $(((\theta_1^1,\Theta_{-i}^1),...,(\theta_1^t,\Theta_{-i}^t)),((s_1^1,.S_{-i}^1),...,(s_1^t,S_{-i}^t)))$ instead of the culmination history $(((\theta_1^1,...,\theta_I^1),...,(\theta_1^t,...,\theta_I^t)),((s_1^1,...,s_I^1),...,(s_1^t,...,s_I^t)))$. The information partition as perceived by agent i with respect to the types of each of the other agents at each stage would be a whole type set $\Theta_j, j \neq i$.

If an information partition of Θ_j as perceived by agent i at stage t is not the whole type set Θ_j , but an informative partition $\{H_j\}^t$, agent i may update her beliefs with regard to the marginal probability of the event θ_j conditional on the information set $h_j^t(\theta_j)$ and all other information available to him at that point of time, including the history as he may perceive. As states in a state space in discussion such as the types in $\Theta_j, j = 1,...,I$ are generally assumed to have positive prior probability with states of probability 0 dropped from a state space, agent i may update her beliefs regarding the marginal probability of the event θ_j conditional merely on the information set $h_j^t(\theta_j)$ simply by forming the conditional probability as $P_{\Theta_j}(\theta_j / h_j^t(\theta_j)) = P_{\Theta_j}(\theta_j) \Big/ \sum_{\theta'_j \in h_j^t(\theta_j)} p_{\Theta_j}(\theta'_j)$. However, belief-updating with regard to unobserved outcomes conditional on such revealed information as histories with time-indexed culmination types and actions would frequently depends not only on assumptions about prior beliefs regarding individual state spaces, but also on equilibrium solution concepts in making inferences about the culmination of a joint type-action outcome. Conditional beliefs play a crucial role in decision-making. In discussing difficulty with marginalization paradoxes for improper priors as noted by Stone and Dawid, Hartigan (1983) considered the case of the uniform distribution over pairs of positive integers $\{i, j\}$, whose desired conditional distribution of $\{i, j\}$ given $j = j_0$ is uniform over i. As Hartigan mentioned, the conditional distribution given j and the marginal distribution should be combined to return to the joint distribution by following $p(i, j) = p(i / j_0)p(j_0)$. The marginal probabilities $p(j_0)$ are not determined by $p(i, j)$, since the event $[\{i, j\}, j = j_0]$ is not given a probability. The problem of belief-updating with respect to unobserved types conditional on histories with observed actions has been tackled with various solution concepts in game-theoretic business research. Among the various solution concepts, the solution concept of perfect Bayesian equilibrium plays a crucial role in the determination of a solution to a problem of incomplete information with unobserved types and observed actions as information-revealing signals.

Based on Bayesian decision theory, the solution concept of perfect Bayesian equilibrium possesses the concepts embedded in the solution concepts of a Bayesian equilibrium and a subgame-perfect equilibrium. As the agents take moves simultaneously for the one-move interaction with private information regarding their own types within an interactive context, each agent may be able to make inferences about the types of the other agents merely conditional on her private information. Within the one-stage interaction, the context as perceived by each agent may be represented symbolically by the measure space $\{\hat{\Omega}, F_{\hat{\Omega}}, p_{\hat{\Omega}}\}$, where $\hat{\Omega} = \left\{((\theta_1,...,\theta_I),(s_1,...,s_I)) \middle| \theta_i \in \Theta_i \wedge s_i \in S_i, i = 1,...,I\right\}$. The belief of agent i about the types of the other agents may be represented by $p_{\hat{\Omega}}\left(\theta_{-i} \middle| \theta_i\right)$. Let the set $S_i^{\Theta_i}$ of maps from Θ_i to S_i be each agent i's space of pure strategies. As mentioned by Fudenburg and Tirole (1991) with regard to a Bayesian equilibrium , $(s_i(\cdot),(s_{-i}(\cdot))$ constitutes a Bayesian equilibrium if $s_i(\cdot)\in \arg\max_{s_i'(\cdot)\in S_i^{\Theta_i}} \sum_{\theta_i}\sum_{\theta_{-i}} p_{\hat{\Omega}}(\theta_i,\theta_{-i})\mu_i(s_i'(\cdot),s_{-i}(\theta_{-i}),(\theta_i,\theta_{-i}))$, $s_i'(\cdot)\in S_i^{\Theta_i}$ for each agent i . Each agent i may use her prior beliefs of the marginal probability $p_{\hat{\Omega}}(\theta_i)$ and the conditional probability $p_{\hat{\Omega}}(\theta_{-i}|\theta_i)$ to derive the probability of $p_{\hat{\Omega}}(\theta_i,\theta_{-i})$. As a type with zero probability would not be considered in decision-making and would thus be irrelevant in analysis, each type under consideration in decision-making would have a positive probability. Therefore, the condition that $s_i(\cdot)\in \arg\max_{s_i'(\cdot)\in S_i^{\Theta_i}} \sum_{\theta_i} p(\theta_i)\sum_{\theta_{-i}} p(\theta_{-i}|\theta_i)\mu_i(s_i'(\cdot),s_{-i}(\theta_{-i}),(\theta_i,\theta_{-i}))$ would enable the *ex ante* formulation for a Bayesian equilibrium $(s_i(\cdot),(s_{-i}(\cdot))$ to be represented equivalently as $s_i(\theta_i)\in \arg\max_{s_i'(\cdot)\in S_i^{\Theta_i}} \sum_{\theta_{-i}} p(\theta_{-i}|\theta_i)\mu_i(s_i'(\cdot),s_{-i}(\theta_{-i})(\theta_i,\theta_{-i}))$ for each $\theta_i \in \Theta_i$.

In the presence of histories with observed actions within a context with multi-stage interaction, an agent may use not merely her private information regarding her own types, but also the information of the observed actions in decision-making. Beliefs-updating within a perfect Bayesian equilibrium would depend on an equilibrium solution concept, processing of the information revealed by observed actions and understanding of equilibrium strategies with the aid of Bayes' rule to make inferences about the unobserved outcomes. As shown by Fudenburg and Tirole (1991) with regard to the solution concept of a perfect Bayesian equilibrium in a two-agent and two-stage model, suppose that agent 1 has private information with regard to her type θ in Θ, which is unobservable to agent 2. In the first stage, player 1 player 1 reveals her action a_1. The culmination history would be (θ,a_1). While agent 2 perceives the action a_1 at the end of stage one, she has no further information except her understanding of the time span $T=2$ and prior beliefs about the unobserved type θ, represented by $p_{\hat{\Omega}}(\theta)$. With the information of the observed action a_1 and her understanding of the mixed strategy $\sigma_1(\cdot/\theta)$ which would be taken by agent 1, agent 2 may update her beliefs at the end of stage 1 from $p_{\hat{\Omega}}(\theta)$ to $p_{\hat{\Omega}}(\theta/a_1)$ by using Bayes' rule as $p_{\hat{\Omega}}(\theta/a_1)=\sigma_1(a_1/\theta)p(\theta)/(\sum_{\theta'}\sigma_1(a_1/\theta')p(\theta'))$. As agent 2 takes a mixed strategy $\sigma_2(\cdot/a_1)$ conditional on the observed action taken by agent 1 in the first stage, the payoff of agent 1 with type θ and a strategy $\sigma_1(\cdot/\theta)$ would be $u_1(\sigma_1,\sigma_{2,},\theta)=\sum_{a_1}\sum_{a_2}\sigma_1(a_1/\theta)\sigma_2(a_2/a_1)u_1(a_1,a_{2,},\theta)$. As agent 1 takes the strategy of $\sigma_1(\cdot/\theta)$, the *ex ante* payoff of player 2 to strategy $\sigma_2(\cdot/a_1)$ when player 1 plays $\sigma_1(\cdot/\theta)$ is $\sum_{\theta} p(\theta)(\sum_{a_1}\sum_{a_2}\sigma_1(a_1/\theta)\sigma_2(a_2/a_1)u_2(a_1,a_{2,},\theta))$. However, with her prior beliefs $p_{\hat{\Omega}}(\theta)$ being updated to $p_{\hat{\Omega}}(\theta/a_1)$ after observing the action a_1 taken by player 1 in the first stage, the *ex post* payoff of player 2 would be $\sum_{a_1} p_{\hat{\Omega}}(a_1)\sum_{\theta} p_{\hat{\Omega}}(\theta / a_1)\sum_{a_2} \sigma_2(a_2 / a_1)u_2(a_1, a_{2,}, \theta)$. If $(\sigma_1^*(\cdot/\theta), \sigma_2^*(\cdot / a_1), p_{\hat{\Omega}}(\theta / a_1))$ constitutes a perfect Bayesian equilibrium, $\forall\theta$, $\sigma_1^*(\cdot / \theta) \in \arg\max_{\sigma_1} u_1(\sigma_1, \sigma_2^*, \theta)$ and $\forall a_1$, $\sigma_2^*(\cdot / a_1) \in \arg\max_{\sigma_2} \sum_{\theta} p_{\hat{\Omega}}(\theta / a_1)u_2(a_1, \sigma_{2,}, \theta)$.

For a perfect Bayesian equilibrium, the strategies are optimal given the beliefs and the beliefs are updated with the information revealed by the strategies. Due to the relation of circularity, a solution to a problem with incomplete information with the aid of the solution concept of PBE is determined by the existence of a set of strategies and beliefs consistent with the circularity relation, and cannot be derived by backward induction. The concept of consistency plays a crucial role in the refined solution concept of a sequential equilibrium proposed by Kreps and Wilson (1982). In the case that for each θ, there exits a unique a_1 such that $\sigma_1^*(a_1 / \theta) = 1$ and $\sigma_1^*(a_1 / \tilde{\theta}) = 0$ for $\tilde{\theta} \neq \theta$, a perfect Bayesian equilibrium would be a separation equilibrium with the posterior beliefs $p_{\hat{\Omega}}(\theta / a_1)$ equal to one. The solution concept of a perfect Bayesian equilibrium in a two-stage and two agent model may be extended to a multi-stage and multi-agent framework under assumptions made about beliefs-updating. Among these are assumptions regarding the prior beliefs about the unobserved types and sequential beliefs-updating for deriving the stage-indexed posterior beliefs. With regard to an outcome space with invariant types $(\theta_1,...,\theta_I)$, the unobserved types are frequently assumed to be independent and the prior distribution $p_{\hat{\Omega}}(\theta_1,...,\theta_I)$ would thus be equal to the product of marginal probabilities $\prod_{i=1}^{I} p_{\hat{\Omega}}(\theta_i)$. With the prior beliefs $p_{\hat{\Omega}}(\theta_i)$ for each θ_i and $p_{\hat{\Omega}}(\theta_1,...,\theta_I)$ for the joint outcome $(\theta_1,...,\theta_I)$ as possessed by each agent, each agent i may update her beliefs about all θ_j after observing the history of $\hat{\omega}_1 = (\theta, a^1)$ at the end of stage 1 by Bayes' rule: $p_{\hat{\Omega}}(\theta_j / (\hat{\omega}^0, a^1) = \sigma_j(a_j^1 / \hat{\omega}^0, \theta_j) p_{\hat{\Omega}}(\theta_j) / (\sum_{\tilde{\theta}_j} \sigma_j(a_j^1 / \hat{\omega}^0, \tilde{\theta}_j) p_{\hat{\Omega}}(\tilde{\theta}_j))$, where $\hat{\omega}^0 = (\phi)$. Beliefs-updating may be made in the successive stages in a similar way as: $p_{\hat{\Omega}}(\theta_j / (\hat{\omega}^{t-1}, a^t) = \sigma_j(a_j^t / \hat{\omega}^{t-1}, \theta_j) p_{\hat{\Omega}}(\theta_j / \hat{\omega}^{t-1}) / (\sum_{\tilde{\theta}_j} \sigma_j(a_j^t / \hat{\omega}^{t-1}, \tilde{\theta}_j) p_{\hat{\Omega}}(\tilde{\theta}_j / \hat{\omega}^{t-1}))$. If the prior beliefs about the types $(\theta_1,...,\theta_I)$ vary among the agents, the beliefs $p_{\hat{\Omega}}(\theta_j)$ for each θ_j may thus be indexed by the agent as $p_{\hat{\Omega}}^i(\theta_j)$ for the prior probability possessed by agent i that agent *j*'s type is θ_j and the prior beliefs $p_{\hat{\Omega}}^i(\theta_1,...,\theta_I)$ would be equal to the product of marginal probabilities $\prod_{j=1}^{I} p_{\hat{\Omega}}^i(\theta_j)$ under the understanding of independent types. With the prior beliefs $p^i_{\hat{\Omega}}(\theta_j)$, each agent i may update her beliefs about all θ_j after observing the culmination history of $\hat{\omega}_1 = (\theta, a^1)$ at the end of stage 1 by Bayes' rule: $p^i_{\hat{\Omega}}(\theta_j /(\hat{\omega}^0, a^1) = \sigma_j(a^1_j / \hat{\omega}^0, \theta_j) p^i_{\hat{\Omega}}(\theta_j) /(\sum_{\tilde{\theta}_j} \sigma_j(a^1_j / \hat{\omega}^0, \tilde{\theta}_j) p^i_{\hat{\Omega}}(\tilde{\theta}_j))$. Each agent i may make beliefs-updating in the successive stages in a similar way by the rule: $p^i_{\hat{\Omega}}(\theta_j /(\hat{\omega}^{t-1}, a^t) = \sigma_j(a^t_j / \hat{\omega}^{t-1}, \theta_j) p^i_{\hat{\Omega}}(\theta_j / \hat{\omega}^{t-1}) /(\sum_{\tilde{\theta}_j} \sigma_j(a^t_j / \hat{\omega}^{t-1}, \tilde{\theta}_j) p^i_{\hat{\Omega}}(\tilde{\theta}_j / \hat{\omega}^{t-1}))$.

Within an interactive managerial context, decision-makers would seek and use relevant information to update beliefs. A firm may acquire information before making an important decision or undertaking a major investment by accessing such external sources of information as marketing, strategic, or information technology consultants, legal advisors, auditors, financial research firms, and information services companies offering electronic services Individuals may also solicit information from various information sources before taking important actions. Sometimes, it would be worthwhile for buyers of used car to invest in a diagnostic check by a trusted mechanic or acquire online information about the history of the vehicle of the specific model they intend to purchase (Arora and Fosfuri, 2005). As mentioned by Gal-Or et al (2006) with regard to information acquisition in advertising industries, companies may present those potential individuals or households with commercials of their interest by using new technologies to better identify those potential customers who would be interested in their products or services. They noted that a personal video recorder (PVR) is programmable and addressable and can record and disclose the viewing patterns and choices of identifiable households. The personal video recorder facilitates companies' construction of demographic profiles of viewers and use of targeted advertising.

By analyzing a perfect Bayesian equilibrium in a product market where consumers know the quality of an incumbent product from its tract record but are uncertain about quality of a new entrant, Balachander (2001) investigated the role warranty would play in quality signaling. As suggested by the study, in response to the offer of a longer warranty by a new entrant an incumbent product may optimally not react with its warranty. Balachander stated that empirical researchers have presented empirical evidence on how warranty and quality are related in a product market but should adjust for differences in consumer knowledge about products in testing the occurrence of signalling with warranties. In contrast to quality signaling by warranty, Stock and Balachander (2005) analyzed marketers' use of product scarcity as a marketing tool in quality signaling in a product market with customers possessing heterogonous information regarding the quality of the product in that later buyers are uninformed consumers in their model. As suggested by their study, unlike a signal produced by price a scarcity strategy affects only the sales from uninformed consumers. Thus, under some conditions signaling through scarcity would be more efficient than price. Bagnoli and Watts (2005) examined managers' decisions on adopting more or less conservative financial reporting policies under generally accepted accounting principles (GAAP) by considering the conditions under which the manager's exercise of discretion may be used by the market to infer her private information about the future prospects and value of the firm. The model developed by

Spence (1974) for the choice of education level is one of the most influential models with quality signaling. In the study of Spence, a worker is attributed with a certain level of ability and decides to choose a level of education. The level of education chosen by a worker would serve as a signal of the worker's ability unobservable to a firm. Among other studies concerning signaling and decision-making are the works of Kreps and Wilson (1982) and Milgrom and Roberts (1982) regarding reputation, predation, and entry deterrence under incomplete information.

6. CONCLUSION

The axioms of the expected-utility maximization theorem assure the existence of a utility function and a conditional-probability function such that choice among a set of lotteries under consideration would be equivalent to choice made by referring to the expected utilities of the prizes associated with the alternative lotteries. Whereas the expected-utility maximization theorem provides a theoretical foundation for analysis of choice under uncertainty, how to represent the beliefs and beliefs-updating associated with the choices of decision-makers within an interactive context appears to be critical in theoretical or empirical research on service-management and other managerial topics, such as pricing of services, quality management and quality signaling, customer relationship management, production and operations management of services, insurance and healthcare management, and contracts and mechanism design. Such influential solution concepts and approaches to dealing with the related issues as discussed in this paper not merely demonstrated the diversified creative endeavors to tackle the problem of beliefs-updating and decision-making, but also signaled the difficulty of the problem and the necessity of further research effort.

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