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Quantum behavior of a two-level atom interacting with two modes of light in a cavity

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A generalized Jaynes-Cummings model is investigated where the transition is mediated by two different modes of photons. For different types (uncorrelated and correlated) of field states, the dynamical behavior of the generalized Jaynes-Cummings model exhibits very different aspects. With the high-Q limit, the effects of cavity damping in the presence of correlated field states are also considered.

I. INTRODUCTION

Recently, a large amount of work have been done on the atomic two-photon transition processes in a cavity both experimentally and theoretically. In the area of experimental work, the most striking event is that Brune *et al.*[1] have successfully made a single-mode two-photon maser operated in a high-Q cavity. This marks an important milestone in quantum optics. The reason is clear since in the past two decades much effort was devoted to establish a practical two-photon laser systems, but no exciting results were given. It was until very recently with the application of the Rydberg-atom microwave-cavity systems that the experimental aspects of testing such kind of problems obtained significant achievement. The success of the single-mode two-photon maser, indeed, relies on the advantages of the Rydberg-atom microwave-cavity systems.[1-3] To advance one step further, we have proposed several schemes for a two-mode two-photon maser with Rydberg-atom microwave-cavity systems in Ref. 4. A remarkable feature of the two-mode two-photon maser is that we can use one mode to modulate, amplify, or control the output of the other mode. In this sense, the Rydberg atom plays a role similar to a coupler which correlates the two modes of light and thus changes the entire photon statistics in a cavity.

As for the theoretical work, Alsing and Zubairy[5] have investigated the collapse and revival of Rabi oscillation of an effective two-level atom which is interacting with a quantized single-mode field through intermediate states. On the same problem, the Stark effects caused by the intermediate states are discussed by Puri and Bullough.[6] Puri and Agarwal[7-9] and Barnett and Knight[10] have developed almost coincidently a technique to deal with a nonideal cavity and have applied it to the case of coherent two-photon transition.[9]

In these existing theoretical works, attention was paid to the detailed discussions on single-mode two-photon processes. There have been no publications concerning the nondegenerate case so far, and this motivates us to study in this paper the generalized Jaynes-Cummings model in which the transition are mediated by two different modes of photons.

The organization of this paper is as follows. In Sec. II some basic formalisms related to the two-photon interaction with one atom are derived based on the approach of Ref. 11. Different types of field states that have been classified into uncorrelated and correlated ones are prepared in Sec. III in which we take two modes of the coherent state and the two-mode squeezed vacuum as the example for each type. With numerical simulations, some results of the dynamical behavior such as the population probability, second-order coherence between the two interacting fields, and the squeezing of the cavity fields are obtained and discussed. Next, in Sec. IV the effects of cavity damping in the presence of the squeezed vacuum are considered by solving the master equation with the high-Q limit. Finally, we conclude the discussions with several conclusions in Sec. V, and the possibility of setting up the experiment is also examined.

II. BASIC FORMALISM

A. Time evolution operator

The system we consider here is an effective two-level atom with upper and lower states denoted by $|1\rangle$ and $|0\rangle$, respectively.

In the two-photon processes, some intermediate states $|i\rangle$, $i=2,3,\ldots$, are involved, which are assumed to be coupled to $|1\rangle$ and $|0\rangle$ by dipole-allowed transitions. Let ω_0, ω_1, and ω_i denote the corresponding frequency of the atomic energy level $|0\rangle$, $|1\rangle$, and $|i\rangle$, respectively. We also denote ω as the transition frequency between states $|1\rangle$ and $|0\rangle$. We start with two requirements: (a) the atom interacts with the two cavity fields with frequencies Ω_1 and Ω_2, where $\Omega_1+\Omega_2\cong\omega$, and (b) $\omega_1-\omega_i$ and $\omega_0-\omega_i$ are off resonance of the one-photon linewidth with Ω_1 and Ω_2. If both are satisfied, then the intermediate states can be adiabatically eliminated[5] and the effective Hamiltonian of the two-level atom can be written in the rotating-wave approximation as

$$\hat{H}=\hat{H}_0+\hat{H}_i\ , \tag{1}$$

with

$$\hat{H}_0=\frac{\omega}{2}\hat{S}_z+(\Omega+\epsilon)\hat{a}_1^\dagger\hat{a}_1+(\Omega-\epsilon)\hat{a}_2^\dagger\hat{a}_2\ , \tag{2}$$

$$\hat{H}_i=g(\hat{a}_1^\dagger\hat{a}_2^\dagger\hat{S}_-+\hat{S}_+\hat{a}_1\hat{a}_2)\ , \tag{3}$$

where $\hat{a}_1^\dagger$ $(\hat{a}_i)$ is the creation (annihilation) operator of cavity modes $i=1,2$,

$$\hat{S}_z=|1\rangle\langle 1|-|0\rangle\langle 0|,\quad \hat{S}_-=|0\rangle\langle 1|,\quad \hat{S}_+=|1\rangle\langle 0| ,$$

Ω is

$$\Omega=\frac{\Omega_1+\Omega_2}{2} ,$$

Ω is the carrier frequency of the two fields, ϵ is

$$\epsilon=\frac{\Omega_1-\Omega_2}{2} ,$$

ϵ is the modulation frequency of the two fields, g is the matrix element of two-photon transition, and Δ is

$$\Delta=\omega-2\Omega .$$

Δ is the detuning.

The Hamiltonian in Eq. (1) is assumed to ignore the Stark shift which might be caused by the intermediate states.

We start our calculation by the standard procedure,[11] namely, the bare-states approach. Since the atom absorbs and emits one pair of photon in an ideal cavity, therefore for a given manifold $\mathcal{M}(n_1,n_2)$ in the bare-states representation, the two bases are $|1,n_1,n_2\rangle$ and $|0,n_1+1,n_2+1\rangle$, which give the form of the free Hamiltonian as

$$\hat{H}_0=\begin{bmatrix} A(n_1,n_2) & 0 \\ 0 & A(n_1,n_2)-\Delta \end{bmatrix} \tag{4}$$

and the interaction Hamiltonian is

$$\hat{H}_i=\begin{bmatrix} 0 & g\sqrt{(n_1+1)(n_2+1)} \\ g\sqrt{(n_1+1)(n_2+1)} & 0 \end{bmatrix} . \tag{5}$$

Thus the total Hamiltonian is

$$\hat{H}=\begin{bmatrix} A(n_1,n_2) & g\sqrt{(n_1+1)(n_2+1)} \\ g\sqrt{(n_1+1)(n_1+1)} & A(n_1,n_2)-\Delta \end{bmatrix} , \tag{6}$$

where $A(n_1,n_2)=(\omega/2)(n_1+n_2+1)-(\Delta/2)(n_1+n_2)+\epsilon(n_1-n_2)$.

The eigenvalues are obtained by solving the secular equation

$$\det\begin{bmatrix} A-\lambda & g\sqrt{(n_1+1)(n_2+1)} \\ g\sqrt{(n_1+1)(n_2+1)} & A-\lambda-\Delta \end{bmatrix}=0 \tag{7}$$

and are found to be

$$\lambda^{\pm}=K(n_1,n_2)\pm Q(n_1,n_2) , \tag{8}$$

where

$$K(n_1,n_2)=\frac{\omega-\Delta}{2}(n_1+n_2+1)+\epsilon(n_1-n_2) ,$$

$$Q(n_1,n_2)=[\Delta^2/4+g^2(n_1+1)(n_2+1)]^{1/2} .$$

The result of Eq. (8) leads to the following matrix representation of the time evolution operator for the given manifold $\mathcal{M}(n_1,n_2)$:

$$\hat{U}(n_1,n_2;t)=e^{-i\hat{H}t}=\frac{1}{2Q}\begin{bmatrix} \left[Q+\frac{\Delta}{2}\right]e^{-i\lambda_+t}+\left[Q-\frac{\Delta}{2}\right]e^{-i\lambda_-t} & g\sqrt{(n_1+1)(n_2+1)}(e^{-i\lambda_+t}-e^{-i\lambda_-t}) \\ g\sqrt{(n_1+1)(n_2+1)}(e^{-i\lambda_+t}-e^{-i\lambda_-t}) & \left[Q-\frac{\Delta}{2}\right]e^{-i\lambda_+t}+\left[Q+\frac{\Delta}{2}\right]e^{-i\lambda_-t} \end{bmatrix} . \tag{9}$$

Once the matrix representation of $\hat{U}(t)$ is obtained, the density operator of the system at time t can be calculated with an arbitrary initial condition $\hat{\rho}(0)$ by

$$\hat{\rho}(t)=\hat{U}(t)\hat{\rho}(0)\hat{U}^{\dagger}(t) . \tag{10}$$

Thus the expectation value of any operator and its dependence on time can be easily obtained through the formula

$$\langle\hat{O}(t)\rangle=\mathrm{Tr}[\hat{\rho}(t)\hat{O}(0)] . \tag{11}$$

B. Population probability and photon distribution

We assume that at time $t=0$ the density operator can be decomposed into its atomic and field parts, i.e.,

$$\hat{\rho}(0)=\hat{\rho}^{A}(0)\otimes\hat{\rho}^{F}(0) , \tag{12}$$

where the atom is in the ith energy eigenstate $|i\rangle$ and the fields are in a general state as

$$|f\rangle=\sum_{n_1,n_2}R_{n_1n_2}|n_1,n_2\rangle . \tag{13}$$

Hence

$$\hat{\rho}(0)=|i,f\rangle\langle i,f|$$

$$=\sum_{n_1',n_2'}\sum_{n_1'',n_2''}R_{n_1'n_2'}R^*_{n_1''n_2''}|i,n_1',n_2'\rangle\langle i,n_1'',n_2''| . \tag{14}$$

Furthermore, we introduce the reduced atomic density operator $\hat{\rho}^{A}(t)$ and the reduced field density operator $\hat{\rho}^{F}(t)$ by taking the trace of $\hat{\rho}(t)$ over the field states and over the atomic states, respectively,

$$\hat{\rho}^{A}(t)=\mathrm{Tr}_F[\hat{\rho}(t)] , \tag{15}$$

$$\hat{\rho}^{F}(t)=\mathrm{Tr}_A[\hat{\rho}(t)] , \tag{16}$$

and their matrix elements are given by

$$\rho^A_{kj}(t)=\sum_{n_1,n_2}\langle k,n_1,n_2|\hat\rho(t)|j,n_1,n_2\rangle\ , \tag{17}$$

$$\rho^F_{n_1',n_2';n_1'',n_2''}=\sum_{k=0}^{1}\langle k,n_1',n_2'|\hat\rho(t)|k,n_1'',n_2''\rangle\ . \tag{18}$$

By means of Eqs. (10) and (12)–(18) we have (i) if $|i\rangle=|1\rangle$

$$\rho^A_{11}(t)=\sum_{n_1,n_2}|R_{n_1n_2}|^2U^\dagger_{11}(n_1,n_2)U_{11}(n_1,n_2)\ , \tag{19}$$

$$\rho^A_{00}(t)=1-\rho^A_{11}(t)\ , \tag{20}$$

$$\rho^A_{10}(t)=\sum_{n_1,n_2}R_{n_1n_2}R^*_{n_1-1\ n_2-1}U^\dagger_{10}(n_1-1,n_2-1)\times U_{11}(n_1,n_2)\ , \tag{21}$$

$$\rho^A_{01}(t)=[\rho^A_{10}(t)]^*\ , \tag{22}$$

(ii) if $|i\rangle=|0\rangle$

$$\rho^A_{00}(t)=\sum_{n_1,n_2}|R_{n_1n_2}|^2U^\dagger_{00}(n_1,n_2)U_{00}(n_1,n_2)\ , \tag{23}$$

$$\rho^A_{11}(t)=1-\rho^A_{00}(t)\ , \tag{24}$$

$$\rho^A_{10}(t)=\sum_{n_1,n_2}R_{n_1+1\ n_2+1}R^*_{n_1n_2}U^\dagger_{00}(n_1,n_2)\times U_{10}(n_1+1,n_2+1)\ , \tag{25}$$

$$\rho^A_{01}(t)=[\rho^A_{10}(t)]^*\ . \tag{26}$$

If we denote $P_{n_1n_2}(0)=|R_{n_1n_2}|^2$ as the initial photon distribution and $P_{n_1n_2}(t)$ as the photon distribution depending upon the time evolution [or the diagonal elements of the reduced field density operator $\hat\rho^F(t)$], then the explicit forms of Eqs. (19)–(26) can be rewritten as (i) if $|i\rangle=|1\rangle$

$$\rho^A_{11}(t)=\sum_{n_1,n_2}P_{n_1n_2}(0)\left[1-\frac{g^2(n_1+1)(n_2+1)\sin^2[Q(n_1,n_2)t]}{Q^2(n_1,n_2)}\right]\ , \tag{27}$$

$$\rho^A_{00}(t)=\sum_{n_1,n_2}P_{n_1n_2}(0)\frac{g^2(n_1+1)(n_2+1)\sin^2[Q(n_1,n_2)t]}{Q^2(n_1,n_2)}\ , \tag{28}$$

$$\rho^A_{10}(t)=\sum_{n_1,n_2}R_{n_1n_2}R^*_{n_1-1\ n_2-1}\frac{g\sqrt{(n_1+1)(n_2+1)}}{2Q^2(n_1,n_2)}\{\Delta\sin^2[Q(n_1,n_2)t]+iQ(n_1,n_2)\sin[2Q(n_1,n_2)t]\}\ , \tag{29}$$

$$\rho^A_{01}(t)=[\rho^A_{10}(t)]^*\ , \tag{30}$$

$$P_{n_1n_2}(t)=P_{n_1n_2}(0)\left[1-\frac{g^2(n_1+1)(n_2+1)\sin^2[Q(n_1,n_2)t]}{Q^2(n_1,n_2)}\right]+P_{n_1-1\ n_2-1}(0)\frac{g^2n_1n_2\sin^2[Q(n_1-1,n_2-1)t]}{Q^2(n_1-1,n_2-1)}\ , \tag{31}$$

(ii) if $|i\rangle=|0\rangle$

$$\rho^A_{00}(t)=\sum_{n_1,n_2}P_{n_1n_2}(0)\left[1-\frac{g^2(n_1+1)(n_2+1)\sin^2[Q(n_1,n_2)t]}{Q^2(n_1,n_2)}\right]\ , \tag{32}$$

$$\rho^A_{11}(t)=\sum_{n_1,n_2}P_{n_1n_2}(0)\frac{g^2(n_1+1)(n_2+1)\sin^2[Q(n_1,n_2)t]}{Q^2(n_1,n_2)}\ , \tag{33}$$

$$\rho^A_{10}(t)=-\sum_{n_1,n_2}R_{n_1+1\ n_2+1}R^*_{n_1n_2}\frac{g\sqrt{(n_1+1)(n_2+1)}}{2Q^2(n_1,n_2)}\{\Delta\sin^2[Q(n_1,n_2)t]+iQ(n_1,n_2)\sin[2Q(n_1,n_2)t]\}\ , \tag{34}$$

$$\rho^A_{01}(t)=[\rho^A_{10}(t)]^*\ , \tag{35}$$

$$P_{n_1n_2}(t)=P_{n_1n_2}(0)\left[1-\frac{g^2n_1n_2\sin^2[Q(n_1-1,n_2-1)t]}{Q^2(n_1-1,n_2-1)}\right]+P_{n_1+1\ n_2+1}(0)\frac{g^2(n_1+1)(n_2+1)\sin^2[Q(n_1,n_2)t]}{Q^2(n_1,n_2)}\ . \tag{36}$$

The probability of finding the atom in its excited state will be denoted as $P_1(t)$ in later discussion.

C. Second-order coherence and squeezing

For convenience, from now on we assume that the system is initially in its excited state $|1\rangle$ and the initial photon states are prepared in two different classes which will be discussed in Sec. III.

We can use the time-dependent photon distribution that is obtained in the previous paragraph to evaluate some quantities relevant to photon statistics. For example the mean photon number of each mode is

$$\langle \hat{n}_1(t)\rangle = \mathrm{Tr}[\hat{\rho}(t)\hat{n}_1] = \sum_{n_1,n_2} P_{n_1n_2}(t)n_1 = \langle \hat{n}_1(0)\rangle + \rho_{00}^A(t) , \quad (37)$$

$$\langle \hat{n}_2(t)\rangle = \mathrm{Tr}[\hat{\rho}(t)\hat{n}_2] = \sum_{n_1,n_2} P_{n_1n_2}(t)n_2 = \langle \hat{n}_2(0)\rangle + \rho_{00}^A(t) . \quad (38)$$

It is obvious that when $t>0$ the field intensity will be enhanced by the two-photon stimulated emission. On the contrary, if the system is initially in its ground state $|0\rangle$, the field intensity will be reduced due to the two-photon stimulated absorption. Also by Eqs. (37) and (38) we can see that the difference in photon numbers between the two modes is a constant which is just the characteristic of the two-photon stimulated emission and absorption.

The expectation value of the correlation of photon numbers for the two modes that is related to the degree of interbeam second-order coherence is

$$\langle \hat{n}_1(t)\hat{n}_2(t)\rangle = \mathrm{Tr}[\hat{\rho}(t)\hat{n}_1\hat{n}_2] = \sum_{n_1,n_2} P_{n_1n_2}(t)n_1n_2 = \langle \hat{n}_1(0)\hat{n}_2(0)\rangle + \sum_{n_1,n_2} P_{n_1n_2}(0)\frac{g^2(n_1+1)(n_2+1)\sin^2[Q(n_1,n_2)t]}{Q^2(n_1,n_2)}(n_1+n_2) + \rho_{00}^A(t) . \quad (39)$$

The degree of interbeam second-order coherence with zero time delay is defined as[12]

$$g_{12}^{(2)} = \frac{\langle \hat{n}_1(t)\hat{n}_2(t)\rangle}{\langle \hat{n}_1(t)\rangle\langle \hat{n}_2(t)\rangle} \quad (40)$$

whose magnitude controls the two-photon transition rate.

Another important property that has not yet been mentioned is the squeezing of the radiation fields in a cavity. Squeezing in Jaynes-Cummings model has been studied by many authors. It has been shown that quadrature squeezing in a high-Q cavity is possible with the coherent light in the Jaynes-Cummings model.[13,14] Drummond[15] has shown that a collection of two-level atoms initially in their ground states would induce squeezing in a coherent single-mode field transiently. By other methods, some single-mode multiphoton generalization of the Jaynes-Cummings model have also been studied[16–19] which exhibit more squeezing in one quadrature phase[18,19] than that of a single photon.

In the two-mode interaction the condition becomes more complicated. The physical variables for describing squeezing in a two-photon device are quadrature-phase amplitudes instead of quadrature phases. With the formalism given by Schumaker and Caves,[20,21] the two quadrature phase amplitudes are defined in the Schrödinger picture as

$$\hat{X}_1(t) = \left[\frac{\Omega+\epsilon}{2\Omega}\right]^{1/2}\hat{a}_1 e^{i\Omega t} + \left[\frac{\Omega-\epsilon}{2\Omega}\right]^{1/2}\hat{a}_2^\dagger e^{-i\Omega t} , \quad (41)$$

$$\hat{X}_2(t) = -i\left[\frac{\Omega+\epsilon}{2\Omega}\right]^{1/2}\hat{a}_1 e^{i\Omega t} + i\left[\frac{\Omega-\epsilon}{2\Omega}\right]^{1/2}\hat{a}_2^\dagger e^{-i\Omega t} , \quad (42)$$

where $\hat{X}_1$ and $\hat{X}_2$ satisfy the following commutation relations:

$$[\hat{X}_1,\hat{X}_1^\dagger] = [\hat{X}_2,\hat{X}_2^\dagger] = \frac{\epsilon}{\Omega} , \quad (43)$$

$$[\hat{X}_1,\hat{X}_2] = 0 , \quad (44)$$

$$[\hat{X}_1,\hat{X}_2^\dagger] = [\hat{X}_1^\dagger,\hat{X}_2] = i . \quad (45)$$

Notice that $\hat{X}_1$ and $\hat{X}_2$ are not Hermitian operators.

In interaction picture the quadrature phase amplitudes are transformed by

$$\hat{X}_m \rightarrow \exp(i\hat{H}_0 t)\hat{X}_m(t)\exp(-i\hat{H}_0 t) . \quad (46)$$

Then, by using Eqs. (41) and (42), one can write the quadrature phases in interaction picture as

$$\hat{E}_m = \Omega^{1/2}(\hat{X}_m e^{-i\epsilon t} + \hat{X}_m^\dagger e^{i\epsilon t}), \quad m=1,2 . \quad (47)$$

Thus the total electric field operator in interaction picture can be written in terms of the quadrature phases with the form

$$\hat{E}(t) = \hat{E}_1(t)\cos(\Omega t) + \hat{E}_2(t)\sin(\Omega t) . \quad (48)$$

However, it is not important what picture we use but instead what information Eqs. (43)–(45) bring to us. To see this it is necessary to define the "reduced" spectral-density matrix[20]

$$\Sigma_{mn}=\frac{\langle \hat{X}_m\hat{X}_n^\dagger+\hat{X}_n\hat{X}_m^\dagger\rangle}{2}-\langle \hat{X}_m\rangle\langle \hat{X}_n^\dagger\rangle , \quad (49)$$

where $m,n=1,2$.

The diagonal elements of Σ_{mn} are simply the mean-square variances of $\hat{X}_1$ and $\hat{X}_2$,

$$\Sigma_{mm}=\langle |\Delta\hat{X}_m|^2\rangle, \quad m=1,2 \quad (50)$$

while the off-diagonal element $\Sigma_{12}=\Sigma_{21}^*$ is a complex correlation term between the two quadratures.

The commutation relations of Eq. (45) imply the uncertainty

$$\langle |\Delta\hat{X}_1|^2\rangle^{1/2}\langle |\Delta\hat{X}_2|^2\rangle^{1/2}\geq \tfrac{1}{2}|\langle[\hat{X}_1,\hat{X}_2^\dagger]\rangle|=\tfrac{1}{2} . \quad (51)$$

If the equality holds at the zero-point, i.e.,

$$\langle |\Delta\hat{X}_1|^2\rangle=\langle |\Delta\hat{X}_2|^2\rangle=\tfrac{1}{2} , \quad (52)$$

then in energy units the total zero-point noise in the two modes is $\frac{1}{2}(\Omega+\epsilon)+\frac{1}{2}(\Omega-\epsilon)=\Omega=\frac{1}{2}(\Omega_1+\Omega_2)$, which means that each quadrature carries one half of the quantum of the zero-point noise. In principle, the uncertainty relation Eq. (51) allows the reduction of noise in one quadrature below the zero-point level with the amplifying of noise in the other quadrature above the zero-point level. In other words, noise can be reduced below the zero-point level only by squeezing noise from one quadrature phase into the other. Thus, by definition, squeezing occurs when $\Delta X_1^2<\frac{1}{2}$ or $\Delta X_2^2<\frac{1}{2}$.

The explicit expressions of the diagonal elements of Σ_{mn} are given according to Eqs. (41), (42), and (49):

$$\begin{aligned}\Sigma_{11}=\frac{1}{2}&+\frac{\Omega+\epsilon}{2\Omega}(\langle \hat{a}_1^\dagger\hat{a}_1\rangle-\langle\hat{a}_1^\dagger\rangle\langle\hat{a}_1\rangle)\\&+\frac{\Omega-\epsilon}{2\Omega}(\langle \hat{a}_2^\dagger\hat{a}_2\rangle-\langle\hat{a}_2^\dagger\rangle\langle\hat{a}_2\rangle)\\&+\frac{(\Omega^2-\epsilon^2)^{1/2}}{2\Omega}[e^{-2i\Omega t}(\langle \hat{a}_1^\dagger\hat{a}_2^\dagger\rangle-\langle\hat{a}_1^\dagger\rangle\langle\hat{a}_2^\dagger\rangle)\\&\quad+\text{c.c.}] ,\end{aligned} \quad (53)$$

$$\begin{aligned}\Sigma_{22}=\frac{1}{2}&+\frac{\Omega+\epsilon}{2\Omega}(\langle \hat{a}_1^\dagger\hat{a}_1\rangle-\langle\hat{a}_1^\dagger\rangle\langle\hat{a}_1\rangle)\\&+\frac{\Omega-\epsilon}{2\Omega}(\langle \hat{a}_2^\dagger\hat{a}_2\rangle-\langle\hat{a}_2^\dagger\rangle\langle\hat{a}_2\rangle)\\&-\frac{(\Omega^2-\epsilon^2)^{1/2}}{2\Omega}[e^{-2i\Omega t}(\langle \hat{a}_1^\dagger\hat{a}_2^\dagger\rangle-\langle\hat{a}_1^\dagger\rangle\langle\hat{a}_2^\dagger\rangle)\\&\quad+\text{c.c.}] .\end{aligned} \quad (54)$$

With Eq. (11) the expectation values of a_1, a_2, and a_1a_2 are calculated as

$$\begin{aligned}\langle\hat{a}_1(t)\rangle&=\text{Tr}[\hat{\rho}(t)\hat{a}_1]\\&=\sum_{n_1,n_2}R_{n_1n_2}R^*_{n_1-1\,n_2}[U_{11}^\dagger(n_1-1,n_2)U_{11}(n_1,n_2)\sqrt{n_1}+U_{10}^\dagger(n_1-1,n_2)U_{01}(n_1,n_2)\sqrt{n_1+1}] ,\end{aligned} \quad (55)$$

$$\begin{aligned}\langle\hat{a}_2(t)\rangle&=\text{Tr}[\hat{\rho}(t)\hat{a}_2]\\&=\sum_{n_1,n_2}R_{n_1n_2}R^*_{n_1n_2-1}[U_{11}^\dagger(n_1,n_2-1)U_{11}(n_1,n_2)\sqrt{n_2}+U_{10}^\dagger(n_1,n_2-1)U_{01}(n_1,n_2)\sqrt{n_2+1}] ,\end{aligned} \quad (56)$$

$$\begin{aligned}\langle\hat{a}_1(t)\hat{a}_2(t)\rangle&=\text{Tr}[\hat{\rho}(t)\hat{a}_1\hat{a}_2]\\&=\sum_{n_1,n_2}R_{n_1n_2}R^*_{n_1-1\,n_2-1}[U_{11}^\dagger(n_1-1,n_2-1)U_{11}(n_1,n_2)\sqrt{n_1n_2}\\&\qquad+U_{10}^\dagger(n_1-1,n_2-1)U_{01}(n_1,n_2)\sqrt{(n_1+1)(n_2+1)}] .\end{aligned} \quad (57)$$

III. PREPARATION OF QUANTUM STATES AND NUMERICAL SIMULATIONS

A. Uncorrelated field states

We use two independent coherent states as the example of uncorrelated field states. A coherent state has the characteristic of Poisson distribution, i.e.,

$$P_n=\frac{e^{-\bar{n}}\bar{n}^n}{n!} , \quad (58)$$

where $\bar{n}$ is the mean photon number.

With two modes of coherent state the coefficient $R_{n_1n_2}$ (assumed to be real) in Eq. (13) can be written as

$$R_{n_1n_2}=\frac{\exp\left[-\frac{\bar{n}_1+\bar{n}_2}{2}\right](\bar{n}_1)^{n_1/2}(\bar{n}_2)^{n_2/2}}{\sqrt{n_1!n_2!}} . \quad (59)$$

The probability of finding the atom in the excited state in the presence of the two modes of coherent state are plotted in Figs. 1 and 2 for various values of detuning. As has been shown in Figs. 1 and 2, the phenomena of quantum revival and quantum collapse of the Rabi oscillation appears. They are more compact than those of one-photon interaction but are not as regular as those in degenerate two-photon case. It is because in the degenerate case, the distribution of the spectral components of the Rabi frequency is one-dimensional since only one

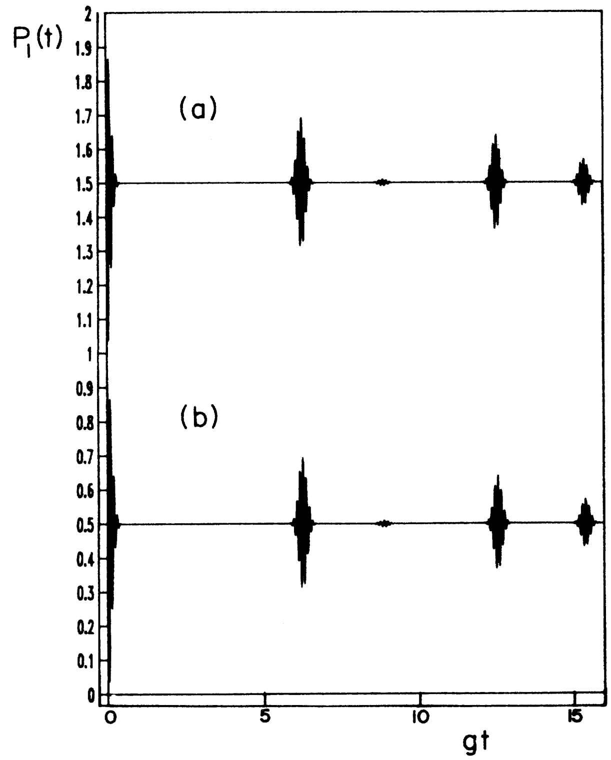

FIG. 1. Probability $P_1(t)$ of finding the atom in the excited state as a function of gt, where the initial field states are prepared in two modes of coherent states with mean photon number $\bar{n}=30$ in each mode; (a) $P_1(t)+1$ with $\Delta=0$, (b) $P_1(t)$ with $\Delta=g$.

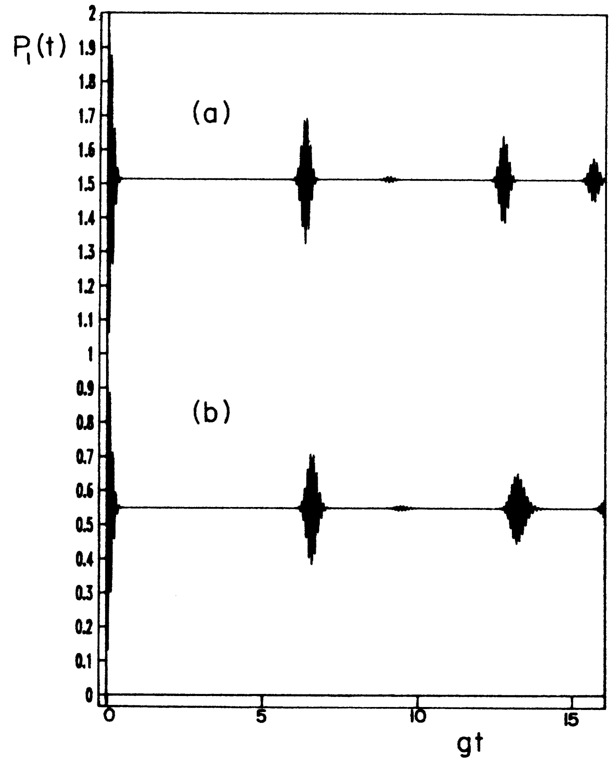

FIG. 2. Probability $P_1(t)$ of finding the atom in the excited state as a function of gt, where the initial field states are prepared in two modes of coherent state with mean photon number $\bar{n}=30$ in each mode; (a) $P_1(t)+1$ with $\Delta=10g$, (b) $P_1(t)$ with $\Delta=20g$.

mode is involved. However, in the nondegenerate case, the distribution of the spectral components of Rabi frequency is two dimensional. In the latter case, each component of Rabi frequency couples the two modes and gives the form of $2[\Delta^2/4+g^2(n_1+1)(n_2+1)]^{1/2}$. With the time-dependent photon distribution, the double summation over the two modes thus causes a certain extent of diffusion among the spectral components of the two modes. As time increases the amplitudes of revival and collapse resulting from the interference of the Rabi frequency will be diminished due to the diffusion of spectral components which also causes the irregularity of the period of the revivals. Figure 3 shows the numerical results of the degree of interbeam second-order coherence. The curves in Fig. 3 exhibit the same form of revival and collapse as the population probability does, but with a certain time delay. Next we study the squeezing of the cavity fields by looking at Figs. 4–6. The variances of the two quadrature phase amplitudes ΔX_1^2 and ΔX_2^2 are plotted in Figs. 4 and 5 as a function of time for various values of mean photon number. We find that squeezing could occur in $\hat{X}_1$, while for $\hat{X}_2$ no squeezing can be observed. The squeezing in $\hat{X}_1$ is transient and we plot it in Fig. 6 in small scale. It seems that the larger photon number induces a larger amount of squeezing. But as the mean photon number increases, the ratio of noise reduction in $\hat{X}_1$ will approach a certain limit which is estimated to be within the range of 32% and 40% in the examples.

B. Correlated field states

The importance of correlated states of light lies in their close connection to the two-photon nonlinear optical processes. Such correlated states also played an important role in quantum mechanics that are involved in certain gedanken experiments which might be helpful in probing some hypotheses of quantum mechanics.

The most important correlated quantum state is the two-mode squeezed state. The degenerate case in the two-mode squeezed state is the familiar one-mode squeezed state which has been a very popular subject recently. The two-mode squeezed state is defined as[21]

$$|\Psi\rangle_{sq}=\hat{D}(\hat{a}_1,\alpha_1)\hat{D}(\hat{a}_2,\alpha_2)\hat{S}(r,\phi)|0\rangle \ , \qquad (60)$$

where $\hat{D}(\hat{a}_i,\alpha_i)$ is the common displacement operator

$$\hat{D}(\hat{a}_i,\alpha_i)=\exp(\alpha_i\hat{a}_i^\dagger-\alpha_i^*\hat{a}_i), \quad i=1,2 \qquad (61)$$

and $\hat{S}(r,\phi)$ is the two-mode squeezing operator

$$\hat{S}(r,\phi)=\exp[r(-\hat{a}_1\hat{a}_2e^{-i\phi}+\hat{a}_1^\dagger\hat{a}_2^\dagger e^{i\phi})] \ . \qquad (62)$$

The expansion of Eq. (60) into number states is quite

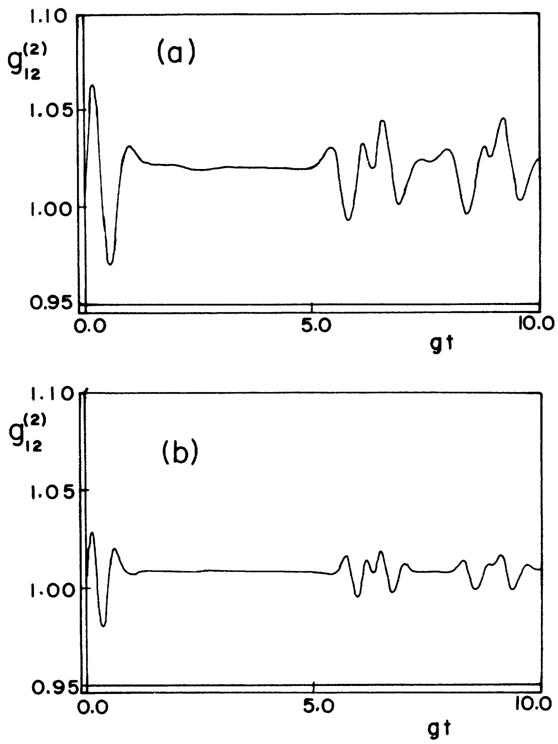

FIG. 3. Degree of interbeam second-order coherence as a function of gt for $\Delta=0$, where the initial field states are prepared in two modes of coherent state with (a) mean photon number $\bar{n}=3$ in each mode, (b) mean photon number $\bar{n}=5$ in each mode.

complicated; however, we would like to consider the simplest form of the two-mode squeezed state,[21,22] i.e., without any displacement ($\alpha_1=\alpha_2=0$):

$$\hat{S}(r,\phi)|0\rangle=(\cosh r)^{-1}\sum_n (e^{i\phi}\tanh r)^n|n,n\rangle \ . \tag{63}$$

It is obvious that squeezed vacuum is a strongly correlated quantum state since in the number-state expansion there are always an equal number of photon in the $\hat{a}_1$ and $\hat{a}_2$ modes.

The probability of finding the atom in the excited state in the presence of squeezed vacuum also exhibits the phenomena of collapse and revival. With little resemblance to the collapses and revivals we have discussed in the preceding paragraph, the revivals are periodically and sharply peaked in the case of zero detuning. It is easy to show that when the detuning Δ is zero, Eq. (27) can be written as

$$P_1(t)=\sum_{n_1,n_2} P_{n_1n_2}(0)\cos^2[Q(n_1,n_2)t] \ . \tag{64}$$

In the case of the two-mode squeezed vacuum, the double summation in Eq. (64) becomes summation over n due to the correlation of the two modes which keeps the equal photon numbers in every mode, i.e.,

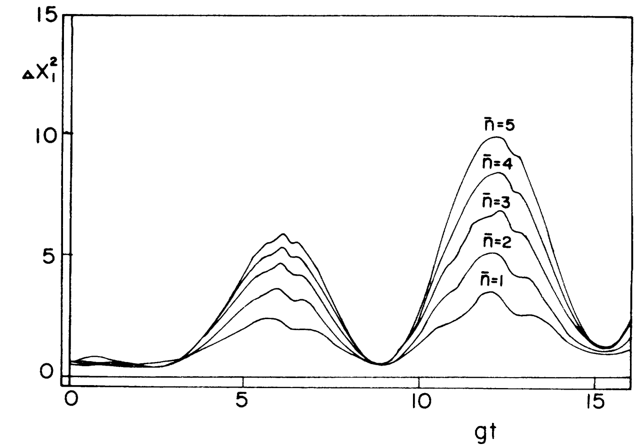

FIG. 4. Variance of $\hat{X}_1$ as a function of gt for various initial mean photon numbers, where the initial field states are prepared in two modes of coherent state ($\bar{n}_1=\bar{n}_2=\bar{n}$ in each mode) with $\Delta=0$, $\Omega=500$ MHz and $\epsilon=5$ MHz.

$$P_{n_1n_2}(0)=\frac{(e^{i\phi}\tanh r)^{2n_1}}{\cosh^2 r}\delta_{n_1,n_2} \ . \tag{65}$$

Thus by Eq. (65) and assuming $\phi=0$, we have

$$P_1(t)=\sum_n \frac{(\tanh r)^{2n}}{\cosh^2 r}\cos^2[g(n+1)t] \ . \tag{66}$$

For a nontrivial periodic solution, $P_1(t+T)=P_1(t)$, we find it is possible that Eq. (66) can always be satisfied only when $T=m\pi/g$, $m=0,1,2,\ldots$. This interesting feature arises from the contribution of the integral factor $n+1$, since for two-photon process in this case the Rabi frequency turns out to be $2g(n+1)$ instead of

$$2g\sqrt{(n_1+1)(n_2+1)}$$

for the previous example or

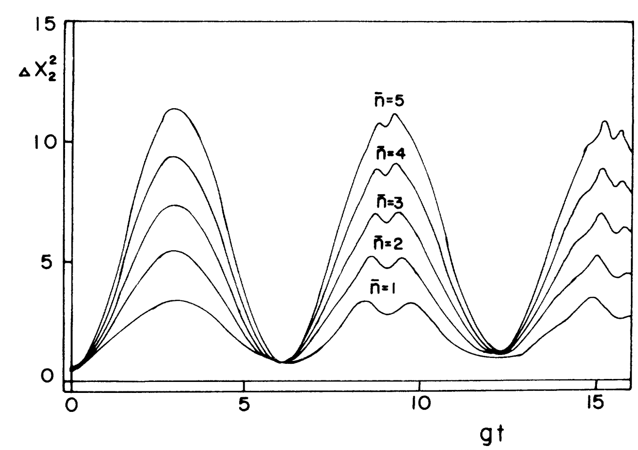

FIG. 5. Variance of $\hat{X}_2$ as a function of gt for various initial mean photon numbers, where the initial field states are prepared in two modes of coherent state ($\bar{n}_1=\bar{n}_2=\bar{n}$ in each mode) with $\Delta=0$, $\Omega=500$ MHz and $\epsilon=5$ MHz.

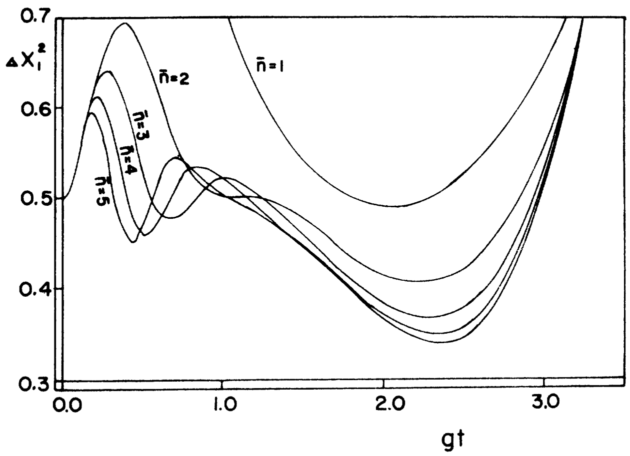

FIG. 6. Maximum squeezing of $\hat{X}_1$ as a function of gt for various initial mean photon numbers, where the initial field states are prepared in two modes of coherent state ($\bar{n}_1=\bar{n}_2=\bar{n}$ in each mode) with $\Delta=0$, $\Omega=500$ MHz, and $\epsilon=5$ MHz.

$$2g\sqrt{(n+1)(n+2)}$$

for the degenerate case. For each n, the period of the corresponding Rabi frequency is $\pi/[g(n+1)]$, for $n=0$ the period becomes π/g, which is the greatest period and is double of the value when $n=1$, triple when $n=2,\ldots,$ and so forth. Hence after doing the summation over all n, the constructive interference will occur with a period of π/g. Unlike the other cases in which the recovering time has dependence on the mean field intensity,[5,9] the recovering time of revival in this case ($\Delta=0$) is thus uniquely determined by the transition-matrix element g. In Fig. 7 for the zero detuning each peak is well located at $t=m\pi/g$, $m=0,1,2,\ldots,$ on the time axis. With increasing detuning the probability of finding the atom in the excited state becomes more and more irregular and has a rate of growing chaotic greater than those of the uncorrelated case in Figs. 1 and 2. It seems, in the case of two-mode squeezed vacuum, the transition mechanism is sensitive to the detuning.

In Fig. 8 we have plotted $g_{12}^{(2)}$ as a function of time for various values of the mean photon number in exact resonance. It is interesting to point out that there are peaks well located around the corresponding transition period $t=m\pi/g$, $m=0,1,2,\ldots,$ which implies that when the stimulated absorption is going to take place, the photon pair will be more strongly correlated at this moment and hence an apparent peak arises around the corresponding period of each revival. Notice that there is a dip at the top of every peak. An explanation for this phenomena is that it might be caused by the high-frequency part of the spectral components. It is easy to verify this. If the upper limit of the summation over the photon number is not large enough, i.e., if we neglect the contribution of the high-frequency part which cannot really by ignored, then we can easily observe the disappears of the dip by computer simulation.

Now, again, let us turn to the squeezing characteristics

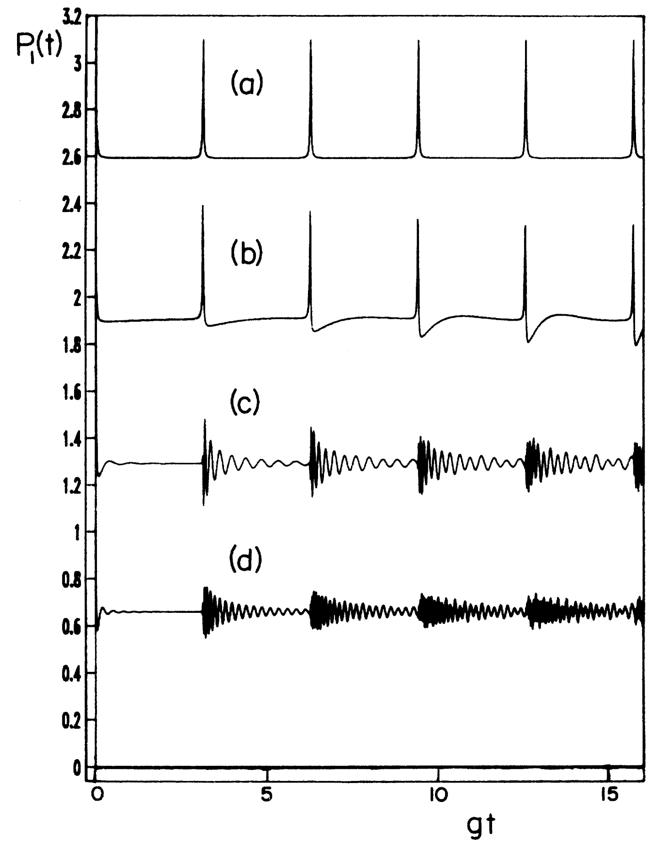

FIG. 7. Probability $P_1(t)$ of finding the atom in the excited state as a function of gt, where the initial field states are prepared in two-mode squeezed vacuum with mean photon number $\bar{n}=\sinh^2 r=30$ in each mode; (a) $P_1(t)+2.1$ with $\Delta=0$, (b) $P_1(t)+1.4$ with $\Delta=g$, (c) $P_1(t)+0.7$ with $\Delta=10g$, (d) $P_1(t)$ with $\Delta=20g$.

of the cavity fields. The numerical results of the variance of the two quadrature phase amplitudes are plotted in Figs. 9 and 10. Unlike the squeezing properties in the previous case, the squeezing occurs in two quadrature phase amplitudes alternatively and periodically. The maximum squeezing in $\hat{X}_1$ is plotted in Fig. 11 as a function of time for various values of mean photon numbers. By Fig. 11, we can find the squeezing much larger than that of Fig. 6. This result agrees with the characteristics of the two-mode squeezed states that have been discussed in Ref. 21.

IV. EFFECTS OF THE CAVITY DAMPING IN THE PRESENCE OF TWO-MODE SQUEEZED VACUUM

In the preceding sections we have restricted out discussion to the ideal case, i.e., without considering the cavity damping. In the realistic situation, a high but finite cavity Q indeed causes some inevitable effects of photon dissipation in the cavity, hence the consideration of cavity damping is supposed to be included in the text.

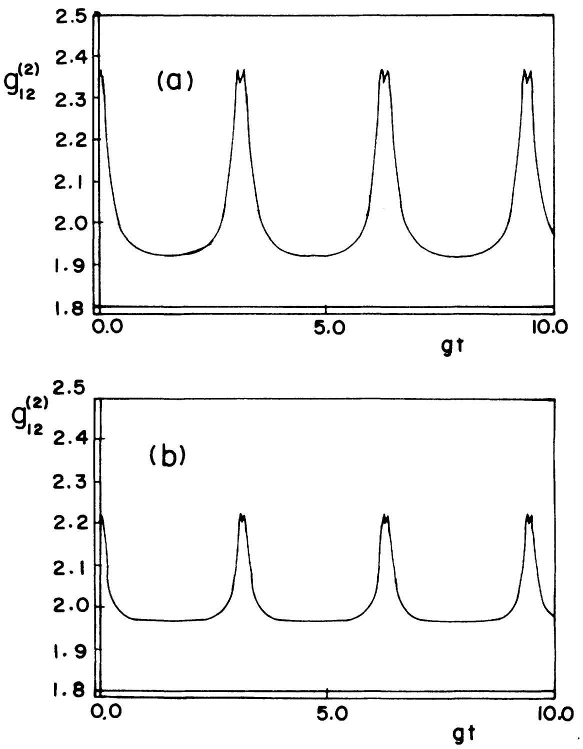

FIG. 8. Degree of interbeam second-order coherence as a function of gt for $\Delta=0$ where the initial field states are prepared in two-mode squeezed vacuum with (a) mean photon number $\bar{n}=\sinh^2 r=3$ in each mode, (b) mean photon number $\bar{n}=\sinh^2 r=5$ in each mode.

Regardless of the thermal photons, a possible way of describing the density matrix for a two-level atom that interacts in exact resonance with two modes of field in a cavity is given by the master equation[23,24]

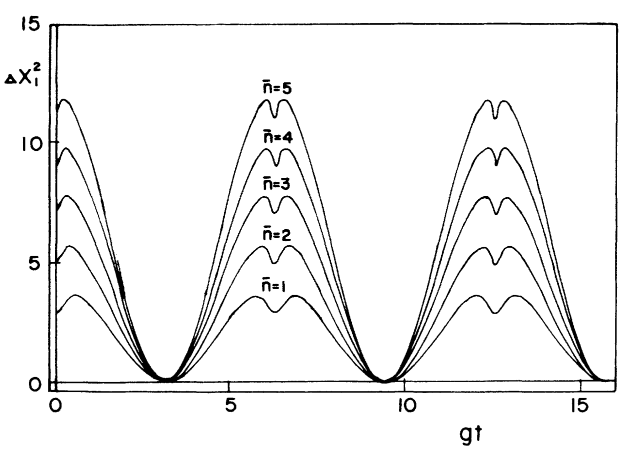

FIG. 9. Variance of $\hat{X}_1$ as a function of gt for various initial photon numbers, which the initial field states are prepared in two-mode squeezed vacuum ($\bar{n}_1=\bar{n}_2=\bar{n}$ in each mode) with $\Delta=0$, $\Omega=500$ MHz, and $\epsilon=5$ MHz.

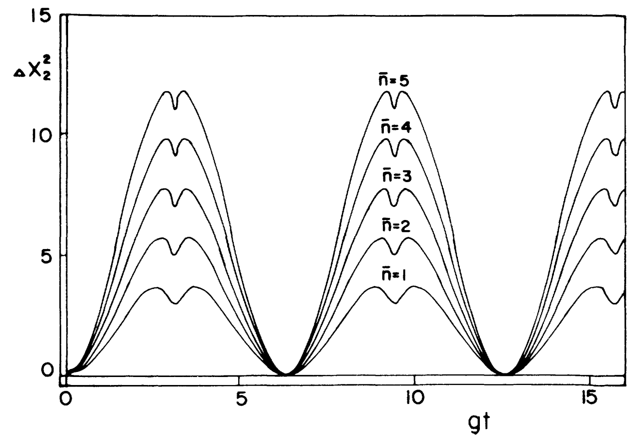

FIG. 10. Variance of $\hat{X}_2$ as a function of gt for various initial photon numbers, where the initial field states are prepared in two-mode squeezed vacuum ($\bar{n}_1=\bar{n}_2=\bar{n}$ in each mode) with $\Delta=0$, $\Omega=500$ MHz, and $\epsilon=5$ MHz.

$$\frac{\partial\hat{\rho}}{\partial t}=-i[\hat{H},\hat{\rho}]-\frac{\kappa}{2}(\hat{a}_1^\dagger\hat{a}_2^\dagger\hat{a}_1\hat{a}_2\hat{\rho}-2\hat{a}_1\hat{a}_2\hat{\rho}\hat{a}_1^\dagger\hat{a}_2^\dagger+\hat{\rho}\hat{a}_1^\dagger\hat{a}_2^\dagger\hat{a}_1\hat{a}_2)\;, \quad (67)$$

where κ is the photon absorption rate of the cavity.

Following the spirit of Puri and Agarwal,[7,8] we now consider the effects of cavity damping in the special case when the fields are initially in the two-mode squeezed vacuum.

To solve Eq. (67) it is easier by using the dressed-state representation than the bare-states representation. The dressed-state representation consists of the complete set of the Hamiltonian including the atom-field interaction, while the bare-states representation consists of the complete set of the uncoupled Hamiltonian. Hence by Eqs. (6), (7), and (8) the eigenstates of $\hat{H}$ are given by

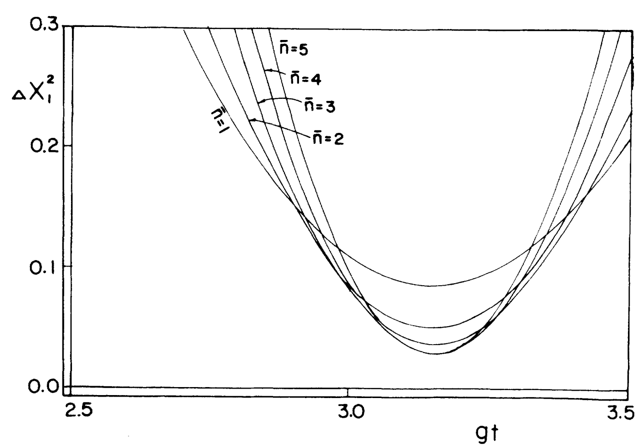

FIG. 11. Maximum squeezing of $\hat{X}_1$ as a function of gt for various initial mean photon numbers, where the initial field states are prepared in two-mode squeezed vacuum ($\bar{n}_1=\bar{n}_2=\bar{n}$ in each mode) with $\Delta=0$, $\Omega=500$ MHz, and $\epsilon=5$ MHz.

$$|\Psi_n^\pm\rangle=\frac{|1,n,n\rangle\pm|0,n+1,n+1\rangle}{\sqrt{2}} \tag{68}$$

with the eigenvalues

$$\lambda_n^\pm=\omega(n+\tfrac{1}{2})\pm g(n+1)\ . \tag{69}$$

Thus

$$\hat{H}|\Psi_n^\pm\rangle=\lambda_n^\pm|\Psi_n^\pm\rangle\ . \tag{70}$$

Now let

$$\hat{W}(t)=e^{i\hat{H}t}\hat{\rho}(t)e^{-i\hat{H}t} \tag{71}$$

and assume that $\kappa \ll g$ (secular approximation), then the equations for the diagonal elements are found to be

$$\langle\Psi_n^\pm|\dot{\hat{W}}(t)|\Psi_n^\pm\rangle=\kappa\left[\frac{(2n+3)^2}{4}\langle\Psi_{n+1}^\pm|\hat{W}(t)|\Psi_{n+1}^\pm\rangle+\tfrac{1}{4}\langle\Psi_{n+1}^\mp|\hat{W}(t)|\Psi_{n+1}^\mp\rangle\right]-\frac{\kappa}{2}[(n+1)^2+n^2]\langle\Psi_n^\pm|\hat{W}(t)|\Psi_n^\pm\rangle\ , \tag{72}$$

$$\langle\Psi_n^\pm|\dot{\hat{W}}(t)|\Psi_n^\mp\rangle=-\kappa\frac{n^2+(n+1)^2}{2}\langle\Psi_n^\pm|\hat{W}(t)|\Psi_n^\mp\rangle. \tag{73}$$

Equation (73) determines the off-diagonal elements of $\hat{W}(t)$,

$$\langle\Psi_n^\pm|\hat{W}(t)|\Psi_n^\mp\rangle=\exp(-G_nt)\langle\Psi_n^\pm|\hat{W}(t)|\Psi_n^\mp\rangle\ , \tag{74}$$

where $G_n=(n+\frac{1}{2})^2+\frac{1}{4}$. To find the diagonal elements of $\hat{W}(t)$ we let

$$F_n(t)=\langle\Psi_n^+|\hat{W}(t)|\Psi_n^+\rangle+\langle\Psi_n^-|\hat{W}(t)|\Psi_n^-\rangle\ , \tag{75}$$

then we can rewrite Eq. (72) as

$$\dot{F}_n(t)=-\kappa G_nF_n(t)+\kappa G_{n+1}F_{n+1}(t)\ . \tag{76}$$

The general solution of Eq. (76) can be written as

$$F_n(t)=\exp(-\kappa G_nt)F_n(0)+\kappa G_{n+1}\int_0^t\exp[-\kappa G_n(t-\tau)]F_{n+1}(\tau)d\tau\ . \tag{77}$$

Formally, the procedure of solving such problems is to assume that the photon distribution initially has a cutoff at N and finally let $N\to\infty$. For the given cutoff at N, the matrix element $\langle\Psi_{N+1}^\pm|\hat{W}(0)|\Psi_{N+1}^\pm\rangle=0$ is always true since the number of photons in the cavity will decrease only. Hence the solution of Eq. (76) is found to be

$$F_N(t)=\exp(-\kappa G_Nt)F_N(0)\ . \tag{78}$$

Using this solution as the driving term for the coupled equation with index $N-1$, all the solution can be obtained by many steps of iteration and for each n has the form

$$F_n(t)=\exp(-\kappa G_nt)\sum_{j=n}^{N}D_j(t)F_j(0)\ . \tag{79}$$

The first few terms of Eq. (79) are given explicitly as

$$D_n(t)=1\ ,$$

$$D_{n+1}(t)=G_{n+1}\frac{1-\exp[-\kappa(G_{n+1}-G_n)t]}{G_{n+1}-G_n}\ ,$$

$$D_{n+2}(t)=\frac{G_{n+2}G_{n+1}}{2(G_{n+2}-G_{n+1})^2}\{1-\exp[-\kappa(G_{n+2}-G_{n+1}]t)\}^2\ ,$$

$$D_{n+3}(t)=\frac{G_{n+1}G_{n+2}G_{n+3}}{2(G_{n+3}-G_{n+2})^2}\left[\frac{1-\exp[-\kappa(G_{n+1}-G_n)t]}{G_{n+1}-G_n}-2\frac{1-\exp[-\kappa(G_{n+3}-G_{n+2}+G_{n+1}-G_n)t]}{(G_{n+3}-G_{n+2}+G_{n+1}-G_n)}+\frac{1-\exp\{-\kappa[2(G_{n+3}-G_{n+2})+G_{n+1}-G_n]t\}}{[2(G_{n+3}-G_{n+2})+G_{n+1}-G_n]}\right]\ ,$$

etc.

Evidently, if $G_{n+1}-G_n=1$ (equivalent to the single-mode case) the preceding expressions will reduce to the simple closed form in Ref. 7. It seems, unfortunately, that Eq. (79) does not possess any closed form. However, it is possible to solve Eq. (76) numerically in finite range of time and we adopt this approach in dealing with the problems. It is reasonable to do so, since within the tolerable error the photon distribution, for example, with initial mean photon number $\bar{n}=5$ when $n=70$ has the value $P_{nn}=4.77\times10^{-7}$, which can be treated as zero.

Now that we are interesting in the population probability of the excited state only, we would like to evaluate $P_1(t)$ in the long run by inverse transformation of Eq. (71). To do this, again we assume that when $t=0$, $\hat{\rho}(0)=\hat{\rho}^A(0)\otimes\hat{\rho}^F(0)$ and the atom is in its excited state, thus in the dressed-state representation we have

$$\hat{W}(0)=\tfrac{1}{2}\sum_{m,n}\chi_{mn}(|\Psi_m^+\rangle\langle\Psi_n^+|+|\Psi_m^-\rangle\langle\Psi_n^-|+|\Psi_m^+\rangle\langle\Psi_n^-|+|\Psi_m^-\rangle\langle\Psi_n^+|)\ . \tag{80}$$

When $m=n$, the coefficient χ_{nn} reduces to the photon distribution P_{nn} of the initial field. Thus we have

$$F_n(0)=\langle\Psi_n^+|\hat{W}(0)|\Psi_n^+\rangle+\langle\Psi_n^-|\hat{W}(0)|\Psi_n^-\rangle=\chi_{nn} \qquad (81)$$

and

$$\langle\Psi_n^+|\hat{W}(0)|\Psi_n^-\rangle=\langle\Psi_n^-|\hat{W}(0)|\Psi_n^+\rangle=\tfrac{1}{2}\chi_{nn} \;. \qquad (82)$$

With Eqs. (71), (79), and (82), it is shown that

$$\begin{aligned}\langle 1,n,n|\hat{\rho}(t)|1,n,n\rangle &=\langle 1,n,n|e^{-i\hat{H}t}\hat{W}(t)e^{i\hat{H}t}|1,n,n\rangle\\ &=\tfrac{1}{2}[F_n(t)+e^{2ig(n+1)t}\langle\Psi_n^+|\hat{W}(t)|\Psi_n^-\rangle+\text{c.c.}]\\ &=\tfrac{1}{2}\exp(-\kappa G_n t)\left[\sum_{j=n}^{N}D_j(t)F_j(0)+P_{nn}\cos[2g(n+1)t]\right] \;. \qquad (83)\end{aligned}$$

The population probability in the excited state is

$$\begin{aligned}P_1(t)=\rho_{11}^A(t)&=\sum_n\langle 1,n,n|\hat{\rho}(t)|1,n,n\rangle\\ &=\tfrac{1}{2}\sum_n\{F_n(t)+P_{nn}\cos[2g(n+1)t]\} \;. \qquad (84)\end{aligned}$$

The numerical results are displayed in Figs. 12 and 13. Just as expected, the cavity damping will smear the amplitudes and spread the width of the revivals. This is because the revivals have dependences on the discrete nature of the photon-number distribution which can be considered as the weighting factors of a discrete spectrum of Rabi frequencies. It is shown by Eq. (83) that weighting factors of higher frequency decay faster than that of lower frequency, that is, the damping of the cavity acts a role somewhat similar to a "low-pass" filter for spectral components of Rabi frequency.

The spreading of the revival reflects the broadening of the spectral components of Rabi frequency. As time increases, when the broadening becomes much larger than the spacing between the Rabi frequencies, the spectrum becomes continuous which will cause the population to collapse and never to revive.[10]

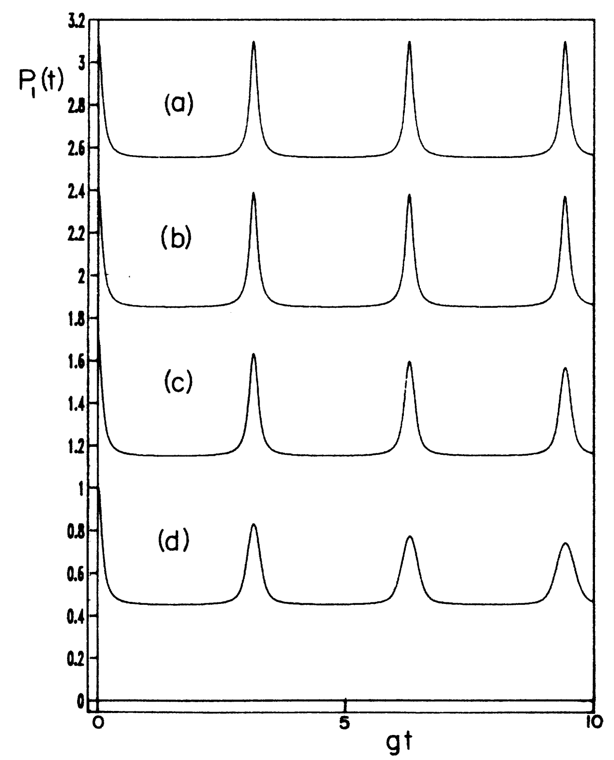

FIG. 12. Probability $P_1(t)$ of finding the atom in the excited state in the presence of cavity damping as a function of gt, where the initial field states are prepared in two-mode squeezed vacuum with mean photon number $\bar{n}=\sinh^2 r=5$ in each mode and $\Delta=0$; (a) $P_1(t)+2.1$ with $\kappa=0$, (b) $P_1(t)+1.4$ with $\kappa/g=0.0001$, (c) $P_1(t)+0.7$ with $\kappa/g=0.001$, (d) $P_1(t)$ with $\kappa/g=0.005$.

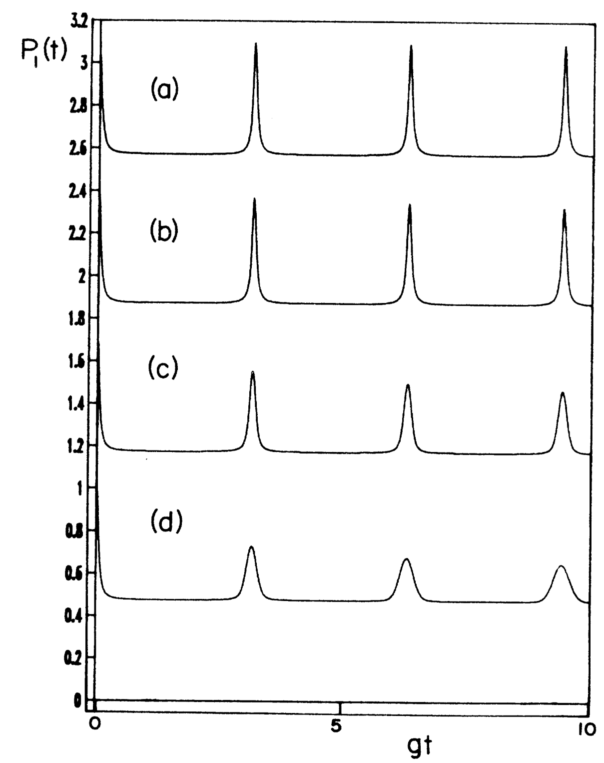

FIG. 13. Probability $P_1(t)$ of finding the atom in the excited state in the presence of cavity damping as a function of gt, where the initial field states are prepared in two-mode squeezed vacuum with mean photon number $\bar{n}=\sinh^2 r=10$ in each mode and $\Delta=0$; (a) $P_1(t)+2.1$ with $\kappa=0$, (b) $P_1(t)+1.4$ with $\kappa/g=0.0001$, (c) $P_1(t)+0.7$ with $\kappa/g=0.001$, (d) $P_1(t)$ with $\kappa/g=0.005$.

V. CONCLUSION

In this paper, we have presented the formalism to describe the interaction of a single atom with two modes of fields in a cavity.

New features of quantum collapse and revival occur in the presence of the two-mode squeezed vacuum. The revivals here appear periodically sharp and thus are able to provide a way to measure the atom-field coupling strength g, or, on the contrary, provide a way to examine whether there is a two-mode squeezed vacuum or not. This novel quantum revival arises from the existence of highly correlated photon pairs which are produced via nonlinear optical processes. The results we have obtained in this paper are somewhat similar to those of Knight.[25] In Knight's paper the atom is prepared in a degenerate three-level atom which leads to another type of two-photon processes.

On the other hand, if the field states are initially in two modes of coherent state, the atom exhibits the phenomena of quantum collapse and revival as well, but no periodicity can be observed evidently. This implies that we can examine the correlation of photon pairs through the interaction of a single atom with two modes of fields.

As for the squeezing of the cavity fields, comparing the correlated and uncorrelated case we find that correlated states induce more squeezing. This agrees with the argument that correlation is responsible for squeezing.

In Sec. IV we have demonstrated the effects of cavity damping which will significantly attenuate the amplitudes of revivals unless the cavity Q is much smaller than the coupling strength g. We did not evaluate the effects of cavity damping in the case of the two modes of the coherent state which can be done in the same manner but more complicated. As for the strong damping, the method used in Sec. IV is no longer available. To treat such problems, the full numerical method is necessary.

To conclude we would like to illustrate some examples which could be helpful for realizing the system we have mentioned.

According to the second-order perturbation theory, the two-photon transition rate is inversely proportional to the detuning δ between the field frequency Ω_1 (or Ω_2) and the transition frequency ω_t that links the initial atomic state to the intermediate state $|i\rangle$.

As Ω_1 (or Ω_2) is tuned cross over the transition frequency ω_t then the one-photon transition $|0\rangle \leftrightarrow |i\rangle$ or $|i\rangle \leftrightarrow |1\rangle$ will dominate over the process. While in coherent two-mode two-photon transition the two requirements (a) and (b) in Sec. II are demanded. Performing in this way, the two-photon oscillator could be available, since in (a) the cavity selects the required resonant modes with fine tuning and provides the environment that the photon pairs emitted by an atom remain stored inside the cavity long enough to be reabsorbed by the same atom with a finite probability, and in (b) if we tune Ω_1 (or Ω_2) very close to ω_t but to an extent which is out of one-photon transition region thus the relative small δ will have a considerable contribution of two-photon gain if suitable atomic states (with large dipole moment) are chosen. An experiment of this kind has been attempted with a He-Ne laser.[26] Being unable to allow Δ to exceed the Doppler width of the one-photon lines, the two-photon gain in this experiment was not large enough to produce significant lasing.

Nevertheless, by the advantages of the Rydberg-atom microwave-cavity systems, we propose in Ref. 4 a scheme of two-mode two-photon maser with such systems. We use the Rydberg states $63P_{1/2}$, $62P_{1/2}$, and $61D_{3/2}$ of the Rb atom as the excited state $|1\rangle$, the ground $|0\rangle$, and the intermediate state $|i\rangle$, respectively.[4] After such an arrangement, the two transition frequencies of $|0\rangle \leftrightarrow |i\rangle$ and $|i\rangle \leftrightarrow |1\rangle$ are 9.591 and 21.111 GHz, respectively, and has a ratio near $\frac{1}{2}$. This series lies in the region of millimeter waves and with such high principal quantum numbers, the dipole moments, $|D_{1i}|$ and $|D_{0i}|$ in this case, are estimated to have the magnitudes about 3000 atomic units.

A practical superconducting cavity has the typical value of Q about 10^9 if the cavity temperature is low enough,[27] thus the cavity linewidth has the magnitude of 10^2 Hz. The Dopper width which depends on the atomic source is assumed to have the magnitude of 200 kHz according to the experimental data.[1-3] The cavity modes can be chosen as long as we need, for example, we may use a rectangular resonant cavity which has the three dimensions of 2.20, 2.20, and 3.40 cm, respectively. The two modes in the cavity are chosen to be TE_{110} and TE_{222} which have the corresponding frequencies of 9.6 and 21.12 GHz. Thus the ratio of δ to ω_t has a value of 9.5×10^{-4} to 4.3×10^{-4}, the transition rate in this estimation is nearly twice that in Ref. 1.

We have ignored the Stark shift in this paper, however, since in the realistic situation the effects of Stark shift become appreciable when δ/ω_t is relatively small. A further discussion on this model including the consideration of Stark shift is planned to be reported elsewhere.

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