Strong Commutativity Preserving Generalized Derivations on Lie Ideals

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Abstract

We apply elementary matrix computations and the theory of differential identities to prove the following: let R be a prime ring with extended centroid C and L a noncommutative Lie ideal of R. Suppose that $f: L \to R$ is a map and g is a generalized derivation of R such that $[f(x), g(y)] = [x, y]$ for all $x, y \in L$. Then there exist a nonzero $\alpha \in C$ and a map $\mu: L \to C$ such that $g(x) = \alpha x$ for all $x \in R$ and $f(x) = \alpha^{-1} x + \mu(x)$ for all $x \in L$, except when $R \subseteq M_2(F)$, the 2×2 matrix ring over a field F.