行政院國家科學委員會專題研究計畫成

RRPA87021015 (3 .P)

反應及擴散自由邊界的一些問題

Reaction and Diffusion Free Boundary Problems

計畫編號：NSC 87-2115-M-018-006

執行期限：86 年 8 月 1 日至 87 年 7 月 31 日

主持人：周忠強　國立彰化師範大學數學系

一、中文摘要

本計劃要探討一類燃燒方程組的一些問題。 此方程組包括了反應擴散方程式及自由邊界等問題，方程組詳細閱敘述於本計劃的緣由與目的中。 於本計劃，首先需要證明方程組的解存在及其唯一性，接著要提出此方程組的數值估計解。

關鍵詞：反應擴散方程式、自由邊界問題、燃燒方程式

Abstract

A set of reaction diffusion free boundary value problem is considered in this report. We shall demonstrate existence and unicity of solutions of the problem.

Keywords: Reaction Diffusion Equations, Free Boundary Problems, Combustion Problems

二、緣由與目的

We shall consider the following reaction diffusion free boundary value problem. Let $z=s(x,y,t)$ denote the free unknown interface between a ''half-space'' full of solid fuel and a ''half-space'' full of gaseous oxidizer and gaseous fuel. Let $z < s(x,y,t)$ denote the gas filled region. Let $u=u(x,y,z,t)$ denote the concentration of the oxidizer and $v=v(x,y,z,t)$ denode the concentration of the gaseous fuel. Let $w=w(x,y,z,t)$ denote the temperature of the gas. The problem considered is that of finding (u,v,w,s) which satisfies

$$
\begin{aligned}
&u_t = \alpha\nabla u + f(u,v), \forall x,y, \text{ and } z < s(x,y,t),\ 0 < t \le T,\\
&v_t = \beta\nabla v + f(u,v), \forall x,y, \text{ and } z < s(x,y,t),\ 0 < t \le T,\\
&w_t = \gamma\nabla w + f(u,v), \forall x,y, \text{ and } z < s(x,y,t),\ 0 \le t \le T,\\
&u(x,y,z,0) = \phi(x,y,z), \forall x,y, \text{ and } z < s(x,y,0),\\
&v(x,y,z,0) = \psi(x,y,z), \forall x,y, \text{ and } z < s(x,y,0),\\
&w(x,y,z,0) = \zeta(x,y,z), \forall x,y, \text{ and } z < s(x,y,0),\\
(1)\quad &\alpha\frac{\partial u}{\partial n} = -\eta\gamma_1(x,y,t) - u(x,y,s(x,y,t),t)\gamma(x,y,t)\\
&\qquad \text{all } x,y, \text{ and } z = s(x,y,t),\ 0 < t \le T,\\
&\beta\frac{\partial v}{\partial n} = \zeta\gamma_0(x,y,t) - v(x,y,s(x,y,t),t)\gamma(x,y,t)\\
&\qquad \text{all } x,y, \text{ and } z = s(x,y,t),\ 0 < t \le T,\\
&\gamma\frac{\partial w}{\partial n} = \mu\gamma_1(x,y,t) - k\gamma_0(x,y,t)\\
&\qquad - w(x,y,s(x,y,t),t)\gamma(x,y,t),\\
&\qquad \text{all } x,y, \text{ and } z = s(x,y,t),\ 0 < t \le T
\end{aligned}
$$

where $u_t \equiv \frac{\partial u}{\partial t}$, etc., ∇ is the Laplacian, n is the outer unit normal to $z=s(x,y,t)$, the fuel evaporation rate γ_0 at $z=s(x,y,t)$ has the form

$$
(2)\quad \gamma_0(x,y,t) = a\exp\{-b/w(x,y,s(x,y,t),t)\},\quad \forall x,y, 0 < t \le T,
$$

the surface oxidation rate γ_1 at $z=s(x,y,t)$ has the form

$$
(3)\quad \gamma_1(x,y,t) = c\exp\{-d/w(x,y,s(x,y,t),t)\}\cdot g(u(x,y,s(x,y,t),t)),
$$

and

$$\gamma(x,y,t)=\gamma_0+\gamma_1. \quad (4)$$

The free boundary $z=s(x,y,t)$ moves under the law

$$s_t(1+(s_x)^2+(s_y)^2=\gamma(x,y,t),$$

$$\forall x,y,0<t\le T, \quad (5)$$

$$s(x,y,0)=s_0(x,y),\forall x,y.$$

Note that all coefficients in above equations are known positive constants, and the functions in above equations are known functions.

We try to demonstrate existence and unicity of solutions of above problem.

三、結果與討論

First, the region described in free boundary conditions of equations (5) in the physical (x, y, z) space is transformed into a fixed region. Then the problem becomes a nonlinear system within a fixed region. It is a classical nonlinear parabolic equations. *Second,* we shall derive some a priori estimates of a solution (u, v, w, s) to the new nonlinear system. *Third,* the main purpose of this project is to demonstrate the following result that "Under suitable assumption, there exists a unique solution (u, v, w, s) to the new problem that depends continuously upon the data.

四、計畫成果自評

The project is not completed yet. There is some difficulty in *Step 3* in section 三 needed to overcome. After it is done, we have some future work. First, numerical methods for the approximation of the solution of the problem will be derived, studies, and implemented for experimental studies. Various appropriate generalizations will be considered in a progressive long term study.

五、參考文獻

[1] A. Bayliss and B. Matkowsky, Fronts, relaxation oscillations and period doubling in solid fuel combustion, J. of Comp. Physics, Vol. 71:147-168.

[2] J. Bebernes and D. Eberly, *Mathematical Problems From Combustion Theory,* Springer-Verlag, New York, 1989.

[3] L. E. Bobisud, Asymptotic dead cores for reaction diffusion equations, J. Math. Anal. Allp., Vol. 147:249-262, 1990.

[4] J. R. Cannon, The One-Dimensional Heat Equation, Encyclopedia of Mathematics, Addison-Wesly, Reading, MA 1984.

[5] J. R. Cannon, J. Cavendish and A. Fasano, A Free boundary value problem related to the combustion of a solid, SIAM J. Appl. Math., Vol. 45:798-809, 1985.

[6] H. K. Chelliah and F. A. Williams, Aspects of the structure of exinction of diffusion flame in methane-oxygen-nitrogen system, Combustion and Flame, Vol. 80:17-48, 1990.

[7] J. Crank, Free and Moving Boundary Problems, Oxford University Press, Oxford, 1984.

[8] G. Dong, Nonlinear Partial Differential Equations of Second Order, Translations of Mathematical Monographs, Vol. 95, the American Mathematical Society, Providence, Rhode Island, 1991.

[9] A. Friedman, Partial Differential Equations of Parabolic Type, Prentice-Hall, Englewood Cliffs, N. J., 1964.

[10] E. S. Oran and J. P. Boris, Numerical

Simulation of Reactive Flow, Elsevier, N. Y., 1987.
[11] P. Ramachandran and L. Doraiswamy, Modeling of noncatalytic gas-solid reactions, AICHE Journal, Vol. 28:881-900, 1982.
[12] F. Shadman and J. C. Cavendish, An analytical model for the combustion of coal particles, Can. J. of Chem. Eng., Vol. 58:470-475, 1980.
[13] G. C. Wake, S. K. Scott and J. Brindley, Multiplicity of steady state solutions for combustion in a solid sphere with arbitrary Biot number, Physics Letters A, Vol. 153: 16-20, 1991.
[14] N. Yanenko, The Method of Fractional Space: the Solutions of Problems of Mathematical Physics in Several Variables, Springer-Verlag, New York, 1971.