行政院國家科學委員會補助專題研究計畫成果報告

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※ 非線性拋物方程式的可調適網格解 ※
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計畫類別：☑個別型計畫　□整合型計畫

計畫編號：NSC89－2115－M－018－013－

執行期間：88 年 08 月 01 日至 89 年 07 月 31 日

計畫主持人：周忠強

共同主持人：

本成果報告包括以下應繳交之附件：

□赴國外出差或研習心得報告一份

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中 華 民 國 89 年 10 月 24 日

行政院國家科學委員會專題研究計畫成果報告

非線性拋物方程式的可調適網格解

Adaptive Solution of Nonlinear Parabolic Equations with Sharp Peaks

計畫編號：NSC 89-2115-M-018-013

執行期限：88 年 8 月 1 日至 89 年 7 月 31 日

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計畫參與人員：xxxxxx　執行機構及單位名稱

一、中文摘要

在此成果報告，我們運用"預測-修正法"在一可調適網格點上解一維非線性拋物方程式，三個主要結果在此報告呈現。首先，在兩個典型例子的測試，我們發現"預測-修正法"比"顯式差分法"表現好地非常多；第二，對於線性拋物方程式的非一致式網格解的收斂性分析，"預測-修正法"比"標準差分法"放寬許多假設條件；最後，在測試的非線性問題上，我們的演算法表現很良好。

關鍵詞：預測-修正法、拋物方程式、收斂性分析、可調適網格點

Abstract

A predictor-corrector method on adaptive grids for one-dimensional parabolic problems is developed in this report. Three issues are investigated for the algorithm. First, we heuristically show that a predictor-corrector method performs much better than explicit difference method by testing on two typical nonlinear examples on uniform grids.
Second, comparing with standard finite difference methods on the analysis of convergence for linear parabolic equations on nonuniform grids, we have found that the predictor-corrector method relaxes more conditions on its assumption. Third, the algorithm performs well on the testing suite.

Keywords: Predictor-Corrector Method, Parabolic Partial Differential Equations, Analysis of Convergence, Adaptive Grids

二、緣由與目的

Many scientific and physical problems have generated systems of time-dependent partial differential equations. Of particular interest to us is a system of parabolic mixed initial-boundary value problem with high spatial activity. The problem is described as:

$$u_{xx} = F(x,t,u,u_x,u_t),\ 0<x<1,\ 0<t<T, \quad (1)$$

$u(x,0), u(0,t), u(1,t)$ specified $0<x<1$, $0<t<T$.

Let w_i^j be the numerical approximations of $u(ih,jk)$ where $0< I< M$, $0<j< J$, $Mh=1$, and $Jk=T$. Further, let

$$x_i=ih,\ t_j=jk,\ \phi_i^j=\phi(x_i,t^j),$$
$$\nabla_x^2\phi_i^j=(\phi_{i+1}^j-2\phi_i^j+\phi_{i-1}^j)/h^2,$$
$$\delta_x\phi_i^j=(\phi_{i+1}^j-\phi_i^j)/2h.$$

Predictor-corrector method was originally considered to numerically solve nonlinear ordinary differential equations. It was proposed by Douglas and Jones in 1963 to apply the idea for solving nonlinear parabolic differential equations. The two difference analgous to solve (1) in [2] can turn algebraic equations linear. Chou and Deng [1] extend the methods to nonlinear system and solve a nonlinear evolution system with small dissipation successfully.

If F has the form

$$F=f_1(x,t,u)u_t+f_2(x,t,u)u_x+f_3(x,t,u),$$

the solution of first difference analgous in [2] converges uniformly to the solution of (1) with error that is $O(h^2+k^2)$.

If F has the form

$$F=g_1(x,t,u,u_x)u_t+g_2(x,t,u,u_x),$$

the solution of second difference analgous in

[2] converges uniformly to the solution of (1) with error that is $O(h^2+k^{3/2})$.

In this report, we investigate the performance of the second analogue of predictor-corrector methods on uniform and nonuniform grids.

三、結果與討論

First, we have two theorems for linear equations.

Theorem 1. (Algorithm 1)
Let $\max_{0\le j\le J} h^j$ and $\max_{0\le j\le J} k^j$ be small enough. The predictor-corrector method is then convergent on uniform grids.

Theorem 2. (Algorithm 2)
Let $\max_{0\le j\le J} h^j$ and $\max_{0\le j\le J} k^j$ be small enough. The predictor-corrector method is then convergent on variable nonuniform grids.

Second, we choose two typical nonlinear examples to test predictor-corrector method. The are the one dimensional Burger's equation [4,5]. We descript them as follows.

$$u_t=\epsilon u_{xx}-uu_x,\ a<x<b,\ t>0, \qquad (2)$$

with the initial condition

$$u(x,0)=\phi(x),\ a<x<b,$$

and the boundary condition

$$u(a,t)=f(t),\ u(b,t)=g(t),$$

where $\epsilon$is a parameter, related to the Reynolds number $R=1/\epsilon$. It is used to be more interesting to solve the Burger's equation with very small ϵ.

Example 1.
The initial and boundary conditions for this example is

$$u(x,0)=\begin{matrix} \sin \pi x, & 0<x\le 1, \\ -(\sin \pi x)/2, & 1<x\le 2, \\ 0, & 2<x<5, \end{matrix}$$

and $u(0,t)=u(5,t)=0$.

Example 2.
The initial and boundary conditions for this example is

$$u(x,0)=\begin{matrix} 1, & 0<x<5, \\ 6-x, & 5\le x\le 2, \\ 0, & 6\le x<12, \end{matrix}$$

and $u(0,t)=1, u(12,t)=0$.

We show our results on figures in the last section after references. From the numerical tests, we have the observations.

1. Comparing Fig. 1 to Fig. 2 and Fig. 5 to Fig. 6, we find out that the number of uniform grids to predictor-corrector method is much less than the number of uniform grids to Forward finite difference method.
2. Comparing Fig. 3 to Fig. 4 and Fig. 7 to Fig. 8, we see that the solutions of Algorithm 2 are better than the solution of Algorithm 1.
3. No matter which adaptive algorithm is employed, it takes less number of grids that uniform grid to approach the solution.

At last, we have two suggestions,

1. For solving nonlinear parabolic equations, predictor-corrector methods are recommended.
2. Adaptive algorithm 2 is fit to solve the problems with high spatial activity, i.e., sharp peaks and steep fronts.

五、參考文獻

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陸、圖表

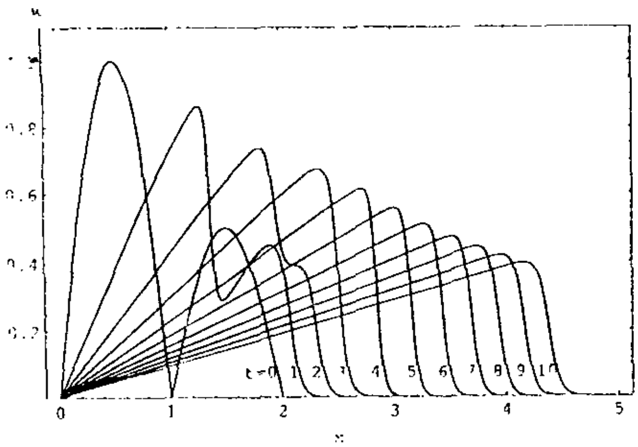


Figure 1. Forward difference method
ε=0.01, Δt=0.00005, 5000 points, Δx=0.001.

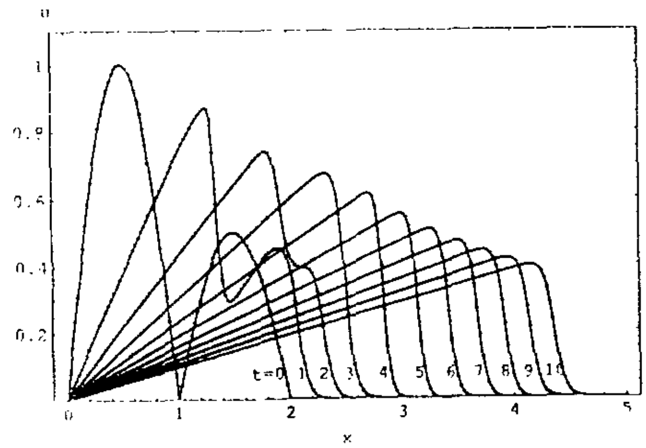


Figure 2. Predictor-corrector method on uniform grids
ε=0.01, Δt=0.02, 200 points, Δx=0.025.

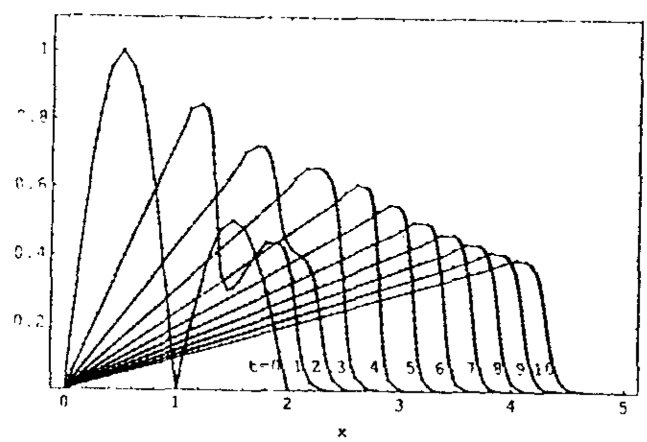
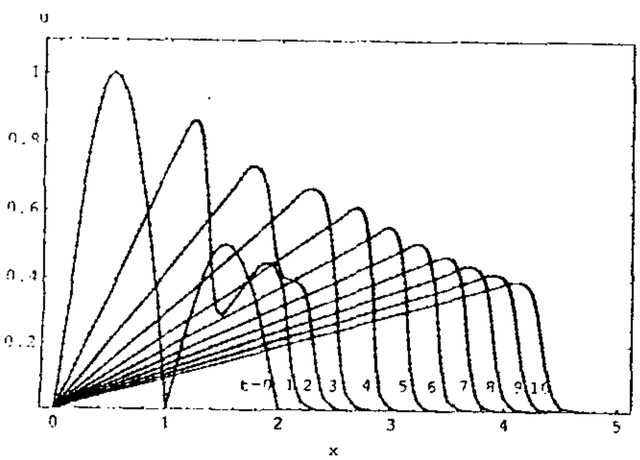


Figure 3. Predictor-corrector method on adaptive grids of algorithm 1
ε=0.01, Δt=0.02, start on 50 points, min:91, max:128, average:103 points.



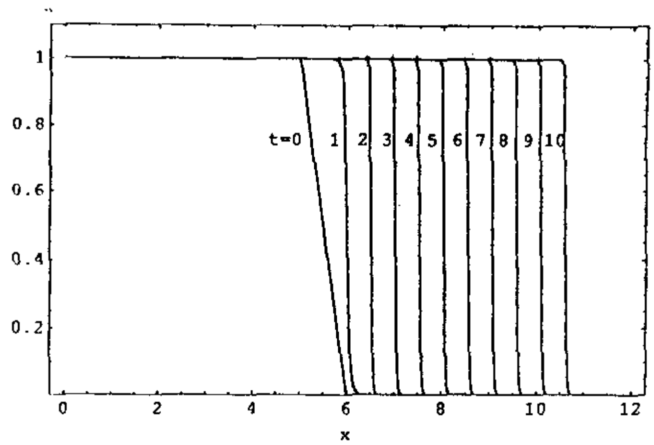


Figure 5. Forward difference method
ε=0.01, Δt=0.00005, 5000 points, Δx=0.0024

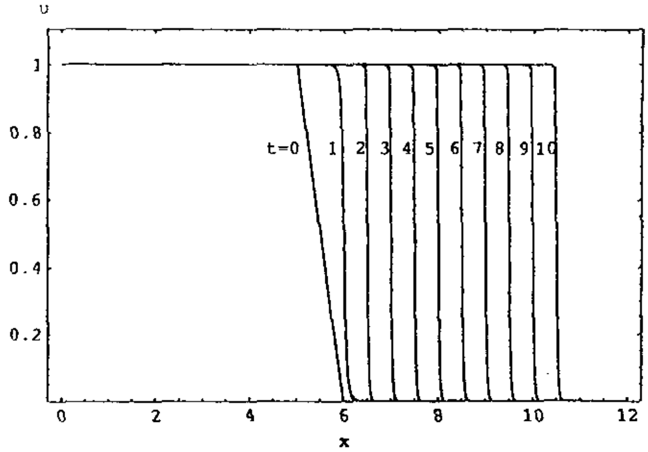


Figure 6. Predictor-corrector method on uniform grids
ε=0.01, Δt=0.02, 500 points, Δx=0.024.

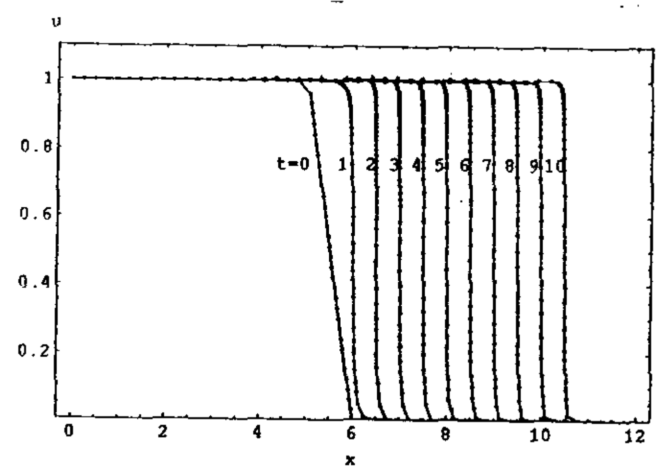


Figure 7. Predictor-corrector method on adaptive grids of algorithm 1
ε=0.01, Δt=0.02, start on 50 points, min:73, max:99, average:88 points.