行政院國家科學委員會補助專題研究計畫成果報告

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※ 可調適網解的穩定性及收斂性分析 ※

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執行期間： 89 年 8月 1日至 90 年 7月 31 日

計畫主持人：周忠強

共同主持人：

計畫參與人員：

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□國際合作研究計畫國外研究報告書一份

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行政院國家科學委員會專題研究計畫成果報告

可調適網解的穩定性及收斂性分析

Stability and Convergence of an Adaptive Technique for Nonlinear Parabolic Equations

計畫編號：NSC 89-2115-M-018-023

執行期限：89 年 8 月 1 日至 90 年 7 月 31 日

主持人：周忠強　國立彰化師範大學數學系

共同主持人：xxxxxx　執行機構及單位名稱

計畫參與人員：xxxxxx　執行機構及單位名稱

一、中文摘要

基本上，本計畫為前一年計劃(89-2115-M-018-013)的延續。於前一個計劃已經成功地發展出可調適網格解，並證明它對於線性拋物線型偏微分方程式俱有穩定及收斂的性質。在此計畫我們證明它對於非線性拋物線方程式亦俱有穩定及收斂的性質。同時也以數個實例來佐證此方法的有效性。

關鍵詞：可調適網格解、非線性拋物線方程式、Burgers 方程式、預測-修正法、穩定性、收斂性

Abstract

This project is a extension of the latest project (89-2115-M-018-013) supported by National Science Council. In the project, we have successfully developed a fully adaptive method for solving one-dimensional parabolic mixed initial-boundary value problems. Also, we proved the stability and convergence of the numerical method on linear parabolic equations. In this project, we probe the stability and convergence of an adaptive technique for nonlinear parabolic equations. In addition, numerical examples are given to exemplify the usefulness of the adaptive technique.

Keywords: Adaptive Methods, Nonlinear Parabolic Partial Differential Equations, Burgers equation, Predictor-Correction Method, Stability, Convergence

二、緣由與目的

Many physical and scientific problems have generated systems of time-dependent partial differential equations. Of particular interest to us is a system of parabolic mixed initial-boundary value problem with high spatial activity. The problems is described as:

$$u_{xx}=F(x,t,u,u_x,u_t),\quad 0<x<1, 0<t<T, \qquad (1)$$

where $u(x,0)$, $u(0,t)$, $u(1,t)$ specified $0<x<1$, $0<t<T$.

Let w_i^j be the numerical approximations of $u(ih,jk)$ where $0<i<M$, $0<j<J$, $Mh=I$, and $Jk=T$. Further, let

$$X_i=ih,\ t^j=jk,\ w_i^j=w(X_i, t^j),$$

$$\Delta_x^2 w_i^j=(w_{i+1}^j-2w_i^j+w_{i-1}^j)/h^2,$$

$$\delta_x w_i^j=(w_{i+1}^j-w_{i-1}^j)/2h.$$

Empolying the Crank-Nicolson difference scheme to (1), obviously it is nonlinear if function F is nonlinear.

Predictor-corrector method was originally considered to numerically solve nonlinear ordinary differential equations. It was proposed by Douglas and Jones in 1963 to apply the idea for solving nonlinear parabolic differential equations. The two difference analgous to solve (1) in [2] can turn algebraic equations linear. Chou and Deng [1] extend the methods to nonlinear system and solve a nonlinear evolution system with small dissipation successfully.

If F has the form

$$F=g_1(x,t,u,u_x)u_t+g_2(x,t,u,u_x),$$

then the analogue has the predictor

$$\Delta_x^2 w_i^{j+1/2}=F\left(X_i, t^{j+1/2}, w_i^j, \delta_x w_i^j, 2(w_i^{j+1/2}-w_i^j)/k\right), \qquad (2)$$

for i=1,2,… ,M-1, followed by the corrector

$$\Delta_x^2 (w_i^{j+1} + w_i^j) = 2F(X_i, t^{j+1/2}, w_i^{j+1/2}, \delta_x w_i^{j+1/2}, (w_i^{j+1} - w_i^j)/k). \quad \textbf{(3)}$$

Clearly the system (2), (3) combined with the initial and boundary data leads to linear algebraic equation. In addition, the solution of the predictor-corrector system (2), (3) converges uniformly to the solution of (1) with an error that is $O(h^2+k^{3/2})$\$.

The strategy for determining a new grid is governed by the inequality

$$|w_i^j - w_{i-1}^j| \leqq \delta \,|\max w_i^j - \min w_{i-1}^j| \qquad \textbf{(4)};$$

a grid point is inserted at the midpoint of the interval if (4) does not satisfy. We can precisely remove points with the opposite strategy. All inserted grid points are ruled by the linear interpolation at the midpoint. The strategy will be explore the benefits from the predictor-corrector method on nonuniform grids by testing on two typical problems.

三、結果與討論

We test our algorithm, based on Predictor-Corrector adaptive method, on the one-dimensional Burgers equation,

$$u_t = \varepsilon u_{tx} - u\, u_x \qquad (5)$$

with the initial condition

$$u(x,0) = \psi(x),\ a < x < b,$$

and the boundary condition

$$u(a,t) = f(t),\ u(b,t) = g(t),$$

where ε is a parameter, related to the Reynolds number $R = 1/\varepsilon$. It is used to be more interesting to solve the burger's equation with very small ε.

Example 1.
The initial and boundary condition for this example is

$$u(x,0) = \begin{cases} 1, & 0 < x < 5, \\ 6-x, & 5 \leqq x < 6, \\ 0, & 6 \leqq x < 12, \end{cases}$$

and $u(0,t) = 1$, $u(12,t) = 0$.

Example 2.
The initial and boundary condition for this example is

$$u(x,0) = \begin{cases} \sin \pi x, & 0 < x < 1, \\ -(\sin \pi x)/2, & 1 \leqq x < 2, \\ 0, & 2 \leqq x < 5, \end{cases}$$

and $u(0,t) = u(5,t) = 0$.

The following figures are used to show our results.

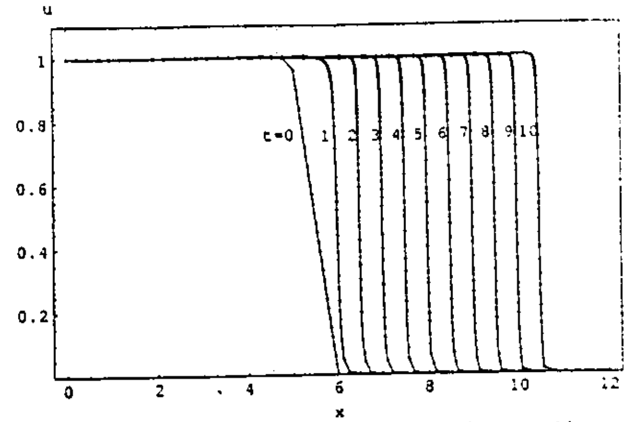

Figure 1. Predictor-corrector method on adaptive grids
ε=0.01, Δt=0.02, start on 50 points, min:73, max:99, average:88 points.

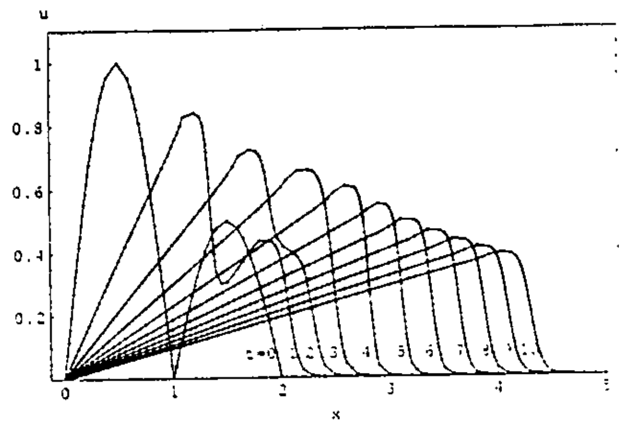

Figure 2. Predictor-corrector method on adaptive grids
ε=0.01, Δt=0.02, start on 50 points, min:91, max:128, average:103 points

From the numerical tests, the adaptive predictor-corrector method capture the solutions efficiently. Especially, it takes a few grids to approach solutions well.

四、計畫成果自評

Since we have very good results on the method. It is supposed to be reorganized to be a peer referred paper. We also plan to develop it on two- or three- dimensional problems.

五、參考文獻

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