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計畫名稱：具有上界曲率空間之幾何不等式探討

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中文摘要

如果 R(Δ)是一個黎曼三角形區域，在一個曲率不大於 1 的空間上，Δ'為Δ的比較三角形在一個單位半球上。並且Δ的周長小於 2π，而且Δ的邊不同倫到一點，則

R(Δ)的面積≦4π-R(Δ')的面積

關鍵詞

Alexandrov 空間，有界曲率空間，CAT(K)空間，同周不等式，幾何不等式，廣義黎曼流型，距離幾何。

On some geometric inequalities on spaces with bounded curvature

Chien-Hsiung Chen

Abstract

Suppose $R(\Delta)$ is a Riemannian triangular region of curvature ≤ 1, and Δ' is its comparison triangle in S^2, with $perim(\Delta) < 2\pi$ and with $\Gamma(\Delta)$ is not homotopic to a point, then we have $area(R(\Delta)) \geq 4\pi - area(R(\Delta'))$. .

Keywords: Alexandrov spaces, spaces with bounded curvature, Cat(K) spaces, isoperimetric inequality, geometric inequality, generalized Riemannian spaces, metric differential geometry.

1 Background and Motivation

The study of metric differential geometry and generalized Riemannian spaces was poineered by A. D. Alexandrov in the 1950's. Generalized Riemannian spaces differ from Riemannian spaces not only in their greater generality but also by the fact they are defined on the basis of their metric alone, without coordinates.

By a space with bounded curvature K in the sense of Alexandrov we means a metric space such that geodesics, angle between two curves can be defined. Suppose that a triangle Twhich is formed by three geodesics in a space with bounded curvature by K, take a corresponding triangle T' such that the side is the same length as T. If the angle of T is not greater than that of corresponding angle in T', than we say that metric space is a space with bounded curvature. It can be proved that A Riemannian manifold with curvature bounded above by K is a space with a bounded curvature from above by K, see [1] for detail.

In this paper we are principally interested in isoperimetric inequalities . The isoperimetric inequalities has a long history, and one could attach to it a long list of names. For an exposition of this, and many related inequalities, we refer to [6]. Here we metion a isoperimetric inequality which is related to our work. The theorem [1]say that the area of a ruled surface spanned on a closed rectifiable curve L in a space with curvature bounded by K with vertex at a point on L is not greater than the area of a disk on space form with length of circle equal to length of L.

2 Main Results and Discussions

By a Riemannian triangular region, we mean a path -metric ρ on the disk, such that $\rho|intD$ is Riemannian, and such that the boundary of D consists of three geodesics arranged in a triangle. The main result of this paper is the following theorem :

Theorem 1 *If$R(\Delta)$ is a Riemannian triangular region of curvature ≤ 1, and Δ' is its comparison triangle in S^2, with $perim(\Delta) < 2\pi$ and with $\Gamma(\Delta)$ is not homotopic to a point, then we have $area(R(\Delta)) \geq 4\pi - area(R(\Delta')$.*

Proof. (Outline) Subdivide $R(\Delta)$ into sufficient small triangles. For each sufficient small triangle the Theorem is true by computation. Then glue all the triangles together. Theorem then follows from Proposition 1 and Proposition 2. □

Proposition 1 *[1] The functional of area has the following properties:*

1. *There is a sequence of complexes that approximate a given surface f in a metric space (M, ρ), the limit of whose areas is the area of the given surface.*

2. *(Semicontinuity) If the sequence of surfaces f_m in (M, ρ) converges uniformly to the surfaces f, then*

$$A(f) \leq \lim_{m\to\infty} A(f_m).$$

3. *(Kolmogrov's principle) If p is a non-expanding map and f is a surface, then $p \circ f$ is a surface in (M, ρ) and*

$$A(p \circ f) \leq A(f).$$

Let (M_1, ρ_1) and (M_2, ρ_2) be metric spaces. We say that a map $f : M_1 \to M_2$ is non-expanding if

$$\rho_2(f(x), f(y)) \leq \rho_1(X, Y), X, Y \in M_1.$$

We say that f, which maps rectifiable curve L_1 in M_1 onto a rectifiable curve L_2 in M_2 , preserves arc length if it takes any arc in L_1 onto an arc of the same length in L_2. Let v be a convex domain in a space with bounded curvature K with bounding curve N. We say that V majorizes a closed rectifiable curve L in a metric space (M, ρ) if there is a non-expanding map of the domain v into M that maps N onto L and preserves arc length.

Proposition 2 *[1] For any closed rectifiable curve L in a space with curvature bounded by K there is a convex domain on a space form with constant curvature K that majorizes the curve L.*

3 Self Evaluation

The main theorem we are proving here is very interesting itself and can be used in minimal area problem in surface theory. The project is quite sucessful because it meets the goal of the auther.

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