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共軛理論在具有螺旋運動機械裝置之幾何建模與特性之研究
Conjugate theory on studying the model and characteristic with screw motion
mechanism

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中文摘要

共軛理論就其性質來說，它是微分幾何理論的範疇，但其具體內容則是一般微分幾何教材中未加論述的，或雖有論述但表達形式不便應用於曲面的共軛理論。共軛理論主要應用於解決機械設計和製造領域裡，諸如確定工具曲面，矯直機軋鋼軋面及空間嚙合曲面等問題的過程中，所發展形成的一門應用學科。

本論文是依據共軛理論、齊次座標轉換原理、微分幾何及電腦動畫模擬，推導一系列具螺旋機械裝置，諸如矯直機軋面、齒輪泵、波紋管及鋼球的軋面數學模型。接著利用所推導的數學模型來建立刀具的路徑和其輪廓幾何模型，然後再利用齒輪共軛嚙合理論來研究創成面和被創成面之間的接觸情形，奠定了矯直軋面輪廓為雙參數或三參數包絡面，才能使被矯直件沿直母線被矯直。再者，本論文亦使用共軛理論於螺旋齒輪泵的裝配誤差研究，確立了中心誤差對螺旋齒輪並無運動誤差的產生。

本論文亦探討所求出的曲面其幾何特性，應用高斯曲率的第一、第二基本齊式的基本量，配合本論文所推導出的曲面數學模型，進行系列的輪廓曲面性質探討，例如曲面的法曲率、主曲率、主方向、一界曲線及二界曲線，這對合成具有良好潤滑油膜厚度的共軛對，具有相當的重要幾何特性。另外，本文亦提供加工過程的刀具路徑規劃及根切分析數學模式、奇異點的產生、軋輪輪廓曲面設計過程及其數學模型以供設計、製造及檢測之重要依據。

雖然這些模型是建立在具有螺旋運動裝置的基礎上，但只要改變一些小程序，便可解決同類一般軋輪及齒輪對製造中相關的問題，更可應用於發展新型的齒輪幾何模型，因此本論文具有廣泛的應用價值。

關鍵字：共軛理論；螺旋運動

Abstract

In terms of principal, conjugate theory is a branch of differential geometry. It is included in differential geometry. But the detail content doesn't find in the books of differential geometry. Little content maybe included in differential geometry. However, their mathematical forms don't conveniently apply to conjugate surface's theory. Conjugate theory mainly applies to solve these questions, such as, machine design, manufacturing, confirmed surface of a tool, straightening roll machine, conjugate surfaces, and so on. In order to solve these questions, the conjugate theory is develop and forms an object of application.

Based on the conjugate theory, homogeneous coordinate transformation, differential geometry, and computer animation, a series of mathematical models of screw motion mechanisms, such as straightening machine, gear pump, multi-wave tubes, and rolling balls are derived. Using the derived mathematical models establishes the cutting tool path, and their geometric models. Based on the conjugate theory and the developed mathematical models investigate the contact condition between generating surface and generated surface. Envelope of the two-parameter family of surfaces or envelope of the three-parameter family of curves can straighten bars along the straightening line. Moreover, in the present dissertation, assembly errors of screw gear pumps are studied by conjugate theory. By the way of analysis, an error of center distance doesn't influence the kinematic error.

In order to investigate the properties of the surface profile, in the present dissertation, the famous Gaussian first and second fundamental forms have been studied. Combined with the results of mathematical models, the geometrical properties which include the normal curvature, the principal curvature, the principal direction, the first limit curves, and the second limit curves are investigate. In order to synthesize the conjugate pair with the better lubrication and form a hydrodynamic lubricating conditions, the analysis of curvatures are necessary and fundamental. Moreover, the cutting tool path, undercutting, the generation of singular points, the design process of rolling contour, and their mathematical models are supplied in the present dissertation. These results can be important in designing, manufacturing, and detecting.

Although the establishment of these models are based on screw motion mechanism. A little change of the process is necessary in solving the similar and relative problems, therefore, it is worthwhile to apply this utensil widely.