CONSUMER PREFERENCES, BUDGET CONSTRAINTS, AND CONSUMPTION CHOICES

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ABSTARCT

This paper discusses consumer preferences and choices from a traditional perspective to illuminate some of the crucial issues relevant to consumer theory and service science. As revealed in the theory of revealed preference, the information which is obtained from the observed consumption choices by consumers may be used by firms in providing and pricing commodities of various types to promote managerial outcomes. The property of transitivity in a preference order relation would facilitate the establishment of the property of coherence in a choice function, while the coherence in a consumer's choice function may not guarantee that her preferences would possess the property of transitivity in a preference order relation. A hypothetical example presented in this paper demonstrates the duality approach to consumer choice analysis by showing the equivalence between utility maximization and expenditure minimization in consumer choice and suggests the necessity of further research on the unaddressed psychological process of individual or group consumption choices made by consumers within a global context.

Keywords: consumer; preference; choice; expenditure; budget constraint

1. INTRODUCTION

A consumer's consumption choice is subject to such crucial factors as her preferences, budget constraints, and external information available to her. As mentioned by Sen (1996), an agent's preferences and choice may be influenced by the specific features of the act of choice, including the identity of the decision maker and the set of items. From the perspective of revenue management, a stringent budget constraint confronting a consumer may constitute one of the prominent identities associated with her and influence considerably the feasible set of items among which a choice is made. In one approach to analyzing a consumer's consumption choice, it is assumed that her preferences may be represented by a utility function. Inspiring in many respects, the study of Afriat (1962) attempted to "give the concepts and the main lines of argument for a theory of the coherence of the expenditure systems, so as to mark out a framework within which there is a definite resolution of some-long-lived enquiries in the abstract theory of the consumer" (p. 305). With incomplete information, the expected-utility maximization theorem is constructed under axioms regarding rational choice behaviors, assuring the existence of a utility function and a conditional-probability function such that the choice made by an agent among a set of lotteries would be equivalent to a choice which is guided by the expected utilities generated under the prizes associated with the alternative lotteries. While Kahneman and Tversky (1979) observed that an agent may not demonstrate a rational choice behavior, in particular underweighting probable outcomes in comparison with certain outcomes, and attempted to develop an alternative theory, the prospect theory, to describe choice under risk, by proposing a value function in terms of a subjective probability measure, termed a decision weight, and a value scale measuring the values of gains or losses to represent an agent's preferences.

While it does not allows for immediate revelation of a consumer's choice under incomplete information, the theory of revealed preference uses the observed choices made by a consumer among commodity bundles to predict her consumption behavior instead of specifying a utility function to represent her preferences for choice analysis. The theory has demonstrated one of the various ways of reasoning which have been developed to describe consumer behavior. This paper discusses some crucial perspectives which have been taken to address issues relevant to consumer preferences, budget constraints, choices, and expenditures, and provides a hypothetical example to illuminate their relationships from the duality perspective.

2. ORDER RELATION AND CHOICE

A budget constraint with which a consumer is confronted determines a feasible class of commodity bundles among which a consumer may choose for consumption. Given a particular set of prices $(p_1^0, p_2^0, ..., p_n^0)$, denoted by p^0, for the n commodities available at the market to a consumer with a budget level y^0, the feasible consumption space would consist of the class $C^0 = \{(q_1, q_2, ..., q_n) \left| \sum_{i=1}^n p_i^0 q_i \le y^0 \right.\}$. With the feasible consumption space, the one-period choice a consumer makes concerns selection from the feasible consumption space C^0. A traditional way of consumer choice analysis adopts an axiomatic set approach. As mentioned by Pinter (1971), the axiom of selection introduced by Zermelo provides one principle for selecting elements in sets and states: "Let A be a set and let $S(x)$ be a statement about x which is meaningful for every object x in A. There exists a set which consists of exactly those elements x in A which satisfy $S(x)$" (p. 10). Furthermore, he described the class axiom in von Neumann's system by stating: "If $S(x)$ is a statement about an object x, there exists a class which consists of all those elements x which satisfy $S(x)$" (p. 13). These two axioms provide one way of thinking associated with a consumer's selection of commodity bundles from feasible consumption spaces and the existence of the selected class.

As stated by Hammond (1976) with regard to a choice set in a set-theoretic framework, an agent, who is confronted with a feasible set of options A and is willing to make choices, can construct a subset $C(A)$ of A as his choice set, from which he is willing to choose any member of it. Furthermore, he mentioned that the choice set $C(A)$ changes as A varies. This would lead to a mapping from the class of feasible sets to the class of choice sets. In the framework of consumer choice, a change in a consumer's budget constraints would lead to a change in the feasible consumption space and thus a change in the corresponding choice set of commodity bundles. A consumer's one-period choice involves the construction of a choice set from the feasible consumption space. As embedded in the selection axiom and the class axiom, the construction of a choice set would need to develop and follow a decision statement $S(x)$, in the terms mentioned above, or a decision rule, which the elements of the choice set would satisfy. From the set-theoretic perspective, consumer choice is analyzed by assuming that a consumer may construct a consumption choice set with the aid of an order relation by applying an order relation to the ordered pairs of the feasible consumption space and undertaking pairwise comparisons of commodity bundles.

As mentioned by Pinter (1971), a binary relation in a class A may be represented by a statement $R(x, y)$ which is either true or false for each ordered pair (x, y) of the class. From the perspective of the representing graph for a relation in a class A, a subclass G of $A \times A$ is a representing graph of a relation in A if it is composed of all the ordered pairs (x, y) such that a statement $R(x, y)$ is true. If a relation is reflexive, anti-symmetric, and transitive, it is called, by definition, an order relation. Thus a relation is an order relation in A if its representing graph G possesses the property that $I_A \subseteq G$, $G \cap G^{-1} \subseteq I_A$, and $G \circ G \subseteq G$, where the diagonal graph $I_A = \{(x, x) | x \in A\}$, $G^{-1} = \{(x, y) | (y, x) \in G\}$, and

$G \circ G = \{(x, y) | \exists z \ni (x, z) \in G \wedge (z, y) \in G\}$.

The order relation plays a crucial role in the construction of a choice function as a mapping from the class of feasible sets to the class of choice sets. As stated by Hammond (1976), with $F(X)$ consisting of all the subsets of the underlying set X , a choice function is a mapping $C : F(X) \to F(X)$, which possesses the property that $C(A) \subseteq A$ for every $A \subseteq X$. For $C : F(X) \to F(X)$ to be a choice function, it would be expected to conform to the choice behaviors which might be observed in choice makers in a context within which a choice is to be made. As described by Hammond with regard to the term of coherence, used by Afriat in describing choices satisfying Houthankker's strong axiom of revealed preference in consumer demand theory, for $A \subseteq B \subseteq X$ and $x \in A - C(A)$, a choice maker may be expected to reject x if the set of options expands from A to B , and a choice function C is coherent if $A - C(A) \subseteq B - C(B)$ for $A \subseteq B \subseteq X$.

As Henderson and Quandt (1980) described regarding consumer choice and revealed preference, the theory of revealed preference attempts to predict a consumer's behavior without the aid of an explicitly specified utility function and provides one way of deriving the characteristics of the utility function of a consumer from the observed choices she makes among commodity bundles. For two commodity bundles q^0 and q^1 such that $p^0 q^1 \le p^0 q^0$ at a price vector p^0, a consumer chooses q^0 rather than q^1, even though q^1 is within the feasible consumption space corresponding to the consumer's budget constraint and the total cost of q^1 is no greater than that of q^0 . The observed consumption choice the consumer makes thus reveals that q^0 is preferred to q^1. By the weak axiom of revealed preference, q^1 would never be revealed to be preferred to q^0 if q^0 is revealed to be preferred to q^1. Suppose that the consumer purchases q^1 instead when the market prices change to p^1. As the observation suggests, the market prices must have changed in a way that

$p^1q^1 < p^1q^0$ and the commodity combination q^0 must have turned out to be unobtainable, since q^1 would never be revealed to be preferred to q^0 by the weak axiom of revealed preference. Furthermore, the strong axiom of revealed preference ensures the transitivity of the preference order relation and asserts that q^k would never be revealed to be preferred to q^0 if q^0 is revealed to be preferred to q^1, q^1 is revealed to be preferred to q^2,…, and q^{k-1} is revealed to be preferred to q^k. The axioms of revealed preference provide one way of deducing consumer's preferences from observed consumption choices. The revealed information which is derived from the observed consumption choices by consumers may be used by firms in providing and pricing commodities of various types to increase gross revenues and net revenues.

3. PREFERENCE TRANSITIVITY AND CHOICE COHERENCE

The axiom of strong revealed preference suggests that the property of transitivity in a preference order relation plays an important role in representing consumer preferences. An analysis of a consumer's choice from the perspective of a choice function or axioms pertinent to a consumer's preferences may share common ways of reasoning. The correspondence between a preference order relation and a choice function may be examined with respect the correspondence between the property of transitivity in a preference order relation and coherence in a choice function. This may be illustrated by examining a triple $\{x, y, z\} \in F(X)$. Let xRy represent the statement that $R(x, y)$ is true. Suppose that a consumer's preferences are characterized by an order relation and conforms to the axiom of transitivity and that zRy and yRx for the set $\{x, y, z\}$. By the property of transitivity, zRy and yRx imply that zRx and (z, x) belongs to the graph of the preference order relation. As Hammond (1976) stated about the correspondence between a preference order relation and a choice function, for all $A \subseteq X$, $x \in C(A)$ implies that xRy for $y \in A$. In terms of a choice function, zRy, yRx, and zRx imply that $C(\{x, y\}) = \{y\}$, and $C(\{y, z\}) = \{z\}$, $C(\{z, x\}) = \{z\}$, and $C(\{x, y, z\}) = \{z\}$. As to the elements which are not chosen in the choice operation on each of these sets, $\{x, y\} - C(\{x, y\}) = \{x\}$, $\{y, z\} - C(\{y, z\}) = \{y\}$, $\{z, x\} - C(\{z, x\}) = \{x\}$, and $\{x, y, z\} - C(\{x, y, z\}) = \{x, y\}$. With regard to the relations between the sets of the elements not chosen, $\{x, y\} - C(\{x, y\}) \subseteq \{x, y, z\} - C(\{x, y, z\})$, $\{y, z\} - C(\{y, z\}) \subseteq \{x, y, z\} - C(\{x, y, z\})$, and $\{z, x\} - C(\{z, x\}) \subseteq \{x, y, z\} - C(\{x, y, z\})$. The property of transitivity in a preference order relation facilitates the establishment of the property of coherence in a choice function.

On the other hand, the coherence in a consumer's choice function may not guarantee that her preferences would possess the property of transitivity in a preference order relation. This may be attributable to the existence of a circular type of choice in a coherent choice function. As stated by Hammond (1976), for a coherent choice function $C : F(X) \to F(X)$, if there exists a set $\{x, y, z\} \in F(X)$ such that $C(\{x, y\}) = \{y\}$, $C(\{y, z\}) = \{z\}$, and $C(\{z, x\}) = \{x\}$, this implies that the choice set $C(\{x, y, z\})$ should be empty. It can be seen that such a circular type of choice in a coherence choice function as Hammond mentioned would correspond to a circular relation in preferences and the property of transitivity in a preference order relation would not be sustained. If the function $C : F(X) \to F(X)$ is a coherent function, then for such a set $\{x, y, z\} \in F(X)$, it should be true that

$\{x,y\}-C(\{x,y\})\subseteq\{x,y,z\}-C(\{x,y,z\})$, $\{y,z\}-C(\{y,z\})\subseteq\{x,y,z\}-C(\{x,y,z\})$, and $\{z,x\}-C(\{z,x\})\subseteq\{x,y,z\}-C(\{x,y,z\})$. For the sets of the elements which are not chosen, $\{x,y\}-C(\{x,y\})=\{x\}$, $\{y,z\}-C(\{y,z\})=\{y\}$, and $\{z,x\}-C(\{z,x\})=\{z\}$. The union of these sets constitutes the triple $\{x,y,z\}$. If the choice set $C(\{x,y,z\})$ is empty, these sets would belong to the set $\{x,y,z\}-C(\{x,y,z\})$ at the same time and the property of coherence in the choice function would be retained. However, in terms of a preference relation, the choice outcome that $C(\{x,y\})=\{y\}$, $C(\{y,z\})=\{z\}$, and $C(\{z,x\})=\{x\}$ implies that yRx zRy, and xRz, which demonstrates a circular relation in preferences and suggests that the property of transitivity is not sustained so far as the feasible set $\{x,y,z\}\in F(X)$ is concerned.

An order relation plays a crucial role in the establishment of the expected-utility maximization theorem. As Myerson (1991) mentioned, the axioms of interest, completeness, and transitivity ensure that an agent would be never indifferent between all the prizes in the set of possible prizes and her preferences are believed to always form a complete transitive order over the set of alternatives. However, in a context with incomplete information within which a choice is to be made by such an agent as a consumer, her preferences and choice may conform to some behavioral axioms. Under the axioms of continuity and monotonicity, a lottery which is ranked between two lotteries would be as good as some randomization between these two and an alternative associated with a higher probability of obtaining a better lottery from a set of lotteries would be better than one associated with a lower probability of acquiring the same lottery. Furthermore, only those possible states would be relevant to the choice by an agent. If an agent must make a choice between two alternatives and she would prefer the one alternative to the other in each event of mutually exclusive events, the substitution axioms state that she must prefer the alternative to the other one even before the culmination of these mutually exclusive events. By the expected-utility maximization theorem, established under these axioms, an agent's choice among a set of lotteries according to an order relation in the set of lotteries would correspond to a choice made by an order relation in the class of the expected utility levels associated with the lotteries in the set of lotteries.

Hammond (1976) mentioned that for a utility function u on the set X, $u(x)\geq u(y)$ if and only if xRy and $C(A)=\{x\in A \mid y\in A\Rightarrow u(x)\geq u(y)\}$ for all $A\subseteq X$. With regard to the correspondence between choice by an order relation in options and that by an order relation in the utilities of options, the statement of Hammond in a framework with complete information is a special version of the expected-utility maximization theorem and reveals how various ways of thinking may contribute to the development of well-established theories. Research efforts have been devoted to this traditional line of research to address issues related not only to individual preferences and choices, but also to their relationships to group or social choices, and have contributed to our understanding of individual or group decision making in such activities as consumption, production, and exchange. Research results in this field are reported in such studies as Batra and Pattanaik (1971), Sen (1969, 1977, 1996), and Steel and Van Der Bellen (1979). Solutions concepts regarding optimal choices within a multi-stage interactive framework with complete or incomplete information are discussed to a thorough extent in Fudenburg and Tirole (1992) and Myerson (1991). Applications of the solution concepts to analysis of optimal choices from the perspective of industrial organization are well presented in

Tirole (1988).

4. CONSUMPTION CHOICE AND EXPENDITURE

While a consumer's choice among commodity bundles may be subject to a variety of factors, the budget constraint confronting her determines a feasible class of commodity bundles among which a consumption choice is to be made. Furthermore, changes in a consumer's preferences and budget levels may take place in a dynamic framework, influencing the feasible consumption space and her evaluation of the satisfaction level derived from a consumption choice. For a firm to develop a revenue-promoting strategy in the provision of a single commodity or multiple commodities within a managerial context with competing commodities available to her consumers, consumers' budget constraints and pursuit of satisfaction levels should be taken into consideration. A traditional duality approach in consumer theory provides an analytical perspective for addressing the issue of price and quality selection. In this section, a hypothetical example will be presented for analysis from a duality perspective to provide information regarding with a consumer's choice in pursuit of a certain utility level with a budget constraint.

Suppose that in a two-commodity framework the utility function of a consumer with a budget level y^0 is represented by $U = q_1 q_2$. Confronted with a consumption budget level y^0 and a market price vector (p_1, p_2), a utility-maximizing consumer is to choose a consumption combination among the feasible consumption class $\{(q_1, q_2) \mid p_1 q_1 + p_2 q_2 \le y^0\}$, denoted by Q^0, by maximizing her utility level. The optimal utility-maximizing consumption combination $(q_1^u, q_2^u) \in C(Q^0)$ would be $((y^0/(2p_1), y^0/(2p_2))$. The utility level derived with the budget level y^0 is $(y^0)^2/(4p_1 p_2)$. For the corresponding minimization problem an expenditure-minimizing consumer needs to tackle, the consumer is to choose a consumption combination among the feasible consumption class $Q^1 = \{(q_1, q_2) \mid U(q_1, q_2) = U^0\}$ by minimizing her expenditure in achieving a targeted utility level U^0. The optimal expenditure-minimizing consumption combination $(q_1^e, q_2^e) \in C(Q^1)$ would be $((U^0 p_2 / p_1)^{1/2}, (U^0 p_1 / p_2)^{1/2})$.

The consumer's choice may be further analyzed by examining the solutions to the dual problem. If the utility level U^0 the expenditure-minimizing consumer pursues is targeted at the utility level of $(y^0)^2/(4p_1 p_2)$, the maximum utility level the consumer may achieve with the consumption budget level y^0, then substituting the targeted utility level $(y^0)^2/(4p_1 p_2)$ for the utility level U^0 in the optimal expenditure-minimizing consumption combination $((U^0 p_2 / p_1)^{1/2}, (U^0 p_1 / p_2)^{1/2})$ would yield $((y^0/(2p_1), y^0/(2p_2))$, which is just the optimal utility-maximizing consumption combination. The minimum expenditure required for obtaining the utility level $(y^0)^2/(4p_1 p_2)$ would be

$p_1 y^0/(2p_1) + p_2 y^0/(2p_2)$, which is just equal to the consumption budget level y^0 given in the utility-maximizing problem. On the other hand, if the consumption budget level the utility-maximizing consumer possesses is $2(U^0 p_2 p_1)^{1/2}$, the minimum expenditure required for obtaining the targeted utility level U^0, then substituting $2(U^0 p_2 p_1)^{1/2}$ for the consumption budget level y^0 in the optimal utility-maximizing consumption combination $((y^0/(2p_1), y^0/(2p_2))$ would yield $((U^0 p_2/p_1)^{1/2}, (U^0 p_1/p_2)^{1/2})$, which is just the optimal expenditure-minimizing consumption combination. The maximum utility level which may be obtained by the utility-maximizing consumer with the consumption budget level $2(U^0 p_2 p_1)^{1/2}$ would be $(U^0 p_2/p_1)^{1/2}(U^0 p_1/p_2)^{1/2}$, which is just equal to the utility level U^0 given in the expenditure-minimizing problem.

While the duality approach to consumer choice analysis provides a mathematical framework within which the equivalence between utility maximization and expenditure minimization in consumer choice can be shown by means of optimization techniques, the results reveal merely parts of the interactions between consumers and firms of various types in a dynamic framework. As described by Cachon and Swinney (2009) with regard to the types of consumers in terms of their purchasing actions in response to prices, bargaining hunting consumers purchase a product only when the price is sufficiently low at a price discount, while myopic consumers purchase only at the initial full price in the first period. In the presence of strategic consumers, whose choice concerns not only purchasing or not purchasing, but also when to purchase, the value of quick response to updated demand information by adjusting inventory, as they found, would be greater to a retailer than without this group of consumers. Furthermore, they argued that a retailer should avoid committing to such a pricing strategy as a markdown price or to not mark down at all. Pricing and quick response to updated demand information are two strategic instruments among the various ones at the disposal of firms to promote their gross and net revenues.

Chambers, Kouvelis, and Semple (2006) examined the market shares and net revenues which may be obtained under price and quality competition by two firms serving a heterogeneous market. They showed under their analytical framework that it would be impossible for a firm with a high commodity quality position to cover the market with a single product and it would be suboptimal for this type of firm to price his commodity to exclude the entire market share from the low-quality firm. In their framework, consumers were characterized by price sensitivity and identical in their sensitivity to quality. Balachander (2001) examined how firms may use warranty to provide customers with information regarding the quality of a product. In the study of Stock and Balachander (2005), consumers' reservation prices for a high-quality commodity are assumed to be distributed uniformly, while those for the low-quality product are taken as identical. They found that only the sales from uninformed consumers would be affected by the use of product scarcity as a tool of quality signaling and thus quality signaling through product scarcity would be more efficient than price under some conditions in their analytical framework. In addressing the issue of recognizing and targeting at potential customers of firms in advertising industries, Gal-Or et al (2006) analyzed targeted advertising and the acquisition of information regarding the demographic profiles of viewers through the use of new technologies to identify and target at those potential customers of companies. These studies have provided information regarding the interaction between consumers and firms from various perspectives.

5. CONCLUSION

In this paper, consumer preferences and choices are discussed from a traditional perspective to illuminate some of the crucial issues relevant to consumer theory and service science. As revealed in the theory of revealed preference, the information which is obtained from the observed consumption choices by consumers may be used by firms in providing and pricing commodities of various types to promote managerial outcomes. The property of transitivity in a preference order relation, as axiom of strong revealed preference suggests, plays an important role in representing consumer preferences. The property of transitivity in a preference order relation facilitates the establishment of the property of coherence in a choice function, while the coherence in a consumer's choice function may not guarantee that her preferences would possess the property of transitivity in a preference order relation. Whereas the duality approach to consumer choice analysis provides a mathematical framework within which the equivalence between utility maximization and expenditure minimization in consumer choice can be shown by means of optimization techniques, the psychological process of individual or group consumption choices made by consumers within a global context deserves further theoretical or empirical study to provide information for global management in service industries.

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