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Fixed Points and Equilibrium Theorems for Better Admissible Multimaps

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ABSTRACT

In this paper we establish some fixed point theorems for a better admissible class of a multimaps in an admissible convex subset of a topological vector space. We also prove many properties of such multimaps and some generalized quasi-equilibrium theorems for them. Based on our results, we can easily obtain some fixed point theorems and equilibria theorems for better admissible multimaps with KKM property.

Key Words: admissible set, better admissible multimap, lower (upper) semicontinuous, Φ-condensing, closed (compact) multimap, acyclic map, convex space, convex hull, *KKM* property, Kakutani (approachable) map, admissible class

I. Introduction

Recently, Chang and Yen (1996) introduced the class $KKM(X,Y)$ that includes most of the multimaps appearing in nonlinear analysis, for example: the Kakutani maps $K(X,Y)$ (Lassonde, 1990), the approachable map $A(X,Y)$ (Ben-El-Mechaiekh and Deguire, 1992) and the admissible class $U_c^k(X,Y)$ (Park, 1994) ect.; for details see Chang (1996) and Chang and Yen (1996). On the other hand, more recently, Park (1997), introduced the better admissible class $\beta(X,Y)$ of multimaps, which is also a large class of multimaps. However, in general, $\beta(X,Y)$ can not be compared with $KKM(X,Y)$.

Moreover, Lin and Yu (1999) established a fixed point theorem for *KKM* type maps on an admissible convex subset in a not-necessary locally convex topological vector space. In this paper, we obtain a fixed point theorem for the better admissible class of multimaps on an admissible convex subset of a topological vector space. Our results also include recent results of Chang and Yen (1996) for the fixed point theorem of the *KKM* type on a locally convex topological vector space.

Based on our results, we deduce fixed point theorems for compact or condensing multimaps in $\beta(X,Y)$ and in some related classes of multimaps. At the same time, we also derive many properties of the better admissible type maps; therefore, generalized quasi-equilibrium theorems are established. Finally, our results extend much work done by Chang and Yen (1996), Lin and Yu (1999) and many others.

II. Preliminaries

Let X, Y be two nonempty sets, and let 2^Y denote the power set of Y. Let $T:X\to 2^Y$ be a set-valued map. $A\subseteq X$, $B\subseteq Y$ and $y\in Y$, and we define

(1) $T(A)=\cup\{T(x): x\in \boldsymbol{A}\}$;
(2) $T^-(y)=\{x\in X: y\in T(x)\}$;
(3) $T^-(B)=\cup\{T^-(y): y\in B\}$;
(4) $g_rT=\{(x,y): y\in T(y),\ x\in X\}$.

For topological spaces X and Y, a map $T:X\to 2^Y$ is said to be closed if g_rT is a closed subset of $X\times Y$; and it is compact if $\overline{T(X)}$ is a compact subset of Y, where $\overline{T(X)}$ is the closure of $T(X)$. T is said to be upper semicontinuous (for short u.s.c.) if the set $T^-(B)$ is closed in X for each closed subset B of Y; it is lower semicontinuous (for short l.s.c.) if the set $T^-(B)$ is open in X for each open subset B of Y; and it is continuous if it is u.s.c. and l.s.c.. If T is u.s.c. with closed values, then T is closed. The converse is true whenever Y is compact (see Aubin and Cellina (1994)).

A convex space X is a nonempty convex set in a linear space with any topology that induces the Euclidean topology on the convex hulls of its finite subsets. For each finite subset A of X, the convex hull of A is called a polytope of X. Note that every convex subset of a topological vector space or of a vector space with the finite topology is a convex space. For details, see Lassonde (1983).

A nonempty subset X of a topological vector space E is said to be admissible (in the sense of Kiee (1960)) provided that, for every compact subset A of X and every neighborhood V of the origin 0 of E, there exists a continuous map $h:A\rightarrow X$ such that $x-h(x)\in V$ for all $x\in A$, and such that $h(A)$ is contained in a finite dimensional subspace L of E. Note that every nonempty convex subset of a locally convex topological vector space is admissible (see Hukuhara (1950) and Nagumo (1951)). Other examples of admissible topological vector spaces are l^P and $L^P(0,1)$ for $0<p<1$, the space $S(0,1)$ of equivalence classes of measurable functions on [0,1], the Hardy space H^P for $0<p<1$, certain Orliez spaces, the ultrabarrelled topological vector space admitting the Schauder basis, and others. Note also that any locally convex subset of an F-normable topological vector space is admissible, and that every compact convex locally convex subset of a topological vector space is admissible. For details, see Park (1997b).

Let E be a topological vector space, and let C be a lattice with a least element, which is denoted by 0. A function $\Phi:2^E\rightarrow C$ is called a measure of noncompactness on E provided that the following conditions hold for any X, $Y\in 2^E$:

(1) $\Phi(X)=0$ if and only if $\overline{X}$ is compact,

(2) $\Phi(\overline{\text{co}}X)=\Phi(X)$, where $\overline{\text{co}}$ denotes the convex closure of X,

(3) $\Phi(X\cup Y)=\max\{\Phi(X),\Phi(Y)\}$.

If $T:X\rightarrow 2^E$, $X\subseteq E$, then T is said to be Φ-condensing provided that if $D\subseteq X$ and $\Phi(D)\leq\Phi(T(D))$, then $\overline{D}$ is compact; that is, $\Phi(D)=0$. Note that every map defined on a compact set or every compact map is Φ-condensing.

Let X and Y be two subsets of the topological vector spaces E and F, respectively. Given two open neighborhoods U and V of the origins in E and F, respectively, a function $s:X\rightarrow Y$ is said to be a (U,V)-approximative continuous selection of $T:X\rightarrow 2^Y$ if for any $x\in X$, $s(x)\in(T[(x+U)\cap X]+V)\cap Y$. A map $T:X\rightarrow 2^Y$ is said to be approachable if its restriction $T|_K$ to any compact subset K of X admits a (U,V)-approximative continuous selection for every U and V as above.

Let W be a class of set-valued maps. We denote $W(X,Y)=\{T:X\rightarrow 2^Y|T\in W\}$, $W_c(X,Y)=\{T_nT_{n-1}\cdots T_1|T_i\in W,\ i=1,2,\cdots,n \text{ for some } n\}$, that is, the set of finite composite of maps in W.

If X and Y are two topological vector spaces, we note that:

(1) $C(X,Y)=\{s:X\rightarrow Y|s$ is a single-valued continuous function$\}$;

(2) $K(X,Y)=\{T:X\rightarrow 2^Y|T$ is u.s.c. with compact convex values and Y is convex$\}$ (see Lassonde (1990));

(3) $A(X,Y)=\{T:X\rightarrow 2^Y|T$ is u.s.c. and approachable with compact values$\}$ (see Ben-El Mechaiekh and Deguire (1992)).

A class U of set-valued maps which was introduced by Park (1994) is one satisfying the following criteria:

(1) U contains the class C of single-valued continuous functions;

(2) each $T\in U_c$ is u.s.c. with compact values;

(3) for each polytope P, each $T\in U_c(P,P)$ has a fixed point.

We denote that $U_c^k(X,Y)=\{T:X\rightarrow 2^Y|$ for any compact subset K of X,and that there is a $F\in U_c(K,Y)$ such that $F(x)\subseteq T(x)$ for each $x\in K\}$.

The following abstract class $KKM(X,Y)$ of set-valued maps was introduced by Chang and Yen (1996). Assume that X is a convex subset of a linear space, and that Y is a topological space. If S, ,T: $X\rightarrow 2^Y$ are two set-valued mappings such that $T(\text{co}A)\subseteq S(A)$ for each finite subset A of X, where co$A$ denotes the convex hull of A, then we call S a generalized KKM mapping w.r.t. T. Let $T:X\rightarrow 2^Y$ be a set-valued mapping such that if $S:X\rightarrow 2^Y$ is a generalized KKM mapping w.r.t. T, then the family $\{\overline{Sx}: x\in X\}$ has the finite intersection property (f.i.p.); then, we say that T has the KKM property. We denote

$$KKM(X,Y)=\{T:X\rightarrow 2^Y|T \text{ has the } KKM \text{ property}\}.$$

Let X be a convex space,and let Y be topological space. Park (1996) first defined the better admissible calss $\beta(X,Y)$ of multimaps as follows :

$$\beta(X,Y)=\{T:x\rightarrow 2^Y|\ \text{for any polytope } P \text{ in } X \text{ and any continuous map } f:T(P)\rightarrow P,\ f(T|_p) \text{ has a fixed point}\}.$$

Throughout this paper, all topological spaces are assumed to be Hausdorff.

III. Main Results

Lemma 1. (Chang and Yen, 1996). Let X be a convex subset of a linear space, and let Y, Z be two topological spaces. Then:

(1) $T\in KKM(X,Y)$ if and only if $T|_p\in KKM(P,Y)$ for each polytope P in X;

(2) if $T \in KKM(X,Y)$ and $f \in C(Y,Z)$, then $fT \in KKM(X,Z)$;
(3) if Y is a normal space, X is a convex space, and P is a polytope in X, and if $T:P \to 2^Y$ is a set-valued mapping such that for each $f \in C(Y,P)$, fT has a fixed point in P, then $T \in KKM(P,Y)$;
(4) if X is a convex space, then $U_c^k(X,Y) \subseteq KKM(X,Y)$.

Lemma 2. (Chang and Yen, 1996). Let X be a convex subset of a locally convex space E, and let $T \in KKM(X,X)$. If T is compact and closed, then T has a fixed point in X.

The following lemma states the relation between $KKM(X,Y)$ and $\beta(X,Y)$.

Lemma 3. (Park, 1997a). If X is a convex space and Y is a topological space, then:
(1) $U_c^k(X,Y) \subseteq (X,Y)$;
(2) if $T \in KKM(X,Y)$ is closed, then $T \in \beta(X,Y)$;
(3) $\beta(X,Y) \subseteq KKM(X,Y)$, if Y is normal;
(4) if $T \in \beta(X,Y)$ is compact, then $T \in KKM(X,Y)$.

Remark 1. From the above lemma, the subclasses $KKM(X,Y)$ and $\beta(X,Y)$ coincide in the class of closed compact maps. However, they may not be comparable in general (Park, 1997a).

We list below some properties of the better admissible class of multimaps.

Theorem 1. Let X be a convex space, and let Y, Z be two topological space. Then:
(1) $T \in \beta(X,Y)$ if and only if $T|_p \in \beta(P,Y)$ for each polytope P in X;
(2) if $T \in \beta(X,Y)$ and $f \in C(Y,Z)$, then $fT \in \beta(X,Z)$;
(3) if A is a convex subset of X and $T \in \beta(X,Y)$, then $T|_A \in \beta(A,Y)$

Proof.
(1) By the definition of $\beta(X,Y)$, the proof is obvious.
(2) For each polytope P in X and $g \in C(fT(P),P)$, it is clear that $gf|_{T(P)} \in C(T(P),P)$. It follows that $gf(T|_p)$ has a fixed point; hence, $g(fT|_p)$ has a fixed point. Thus, $fT \in \beta(X,Z)$.
(3) Let P be any polytope in A. Since $T \in \beta(X,Y)$, it follows from (1) that $T|_p \in \beta(P,Y)$. However, $(T|_A)|_P = T|_p \in \beta(P,Y)$. Again, by applying (1), $T|_p \in \beta(A,Y)$. □

By the result of Lin and Yu (1999), we can establish some fixed point theorems for the better admissible class of multimaps on an admissible convex subset of a topological vector space.

Lemma 4. (Lin and Yu, 1999). Let E be a topological vector space, let X be an admissible convex subset of E, and let $F \in KKM(X,X)$ be compact and closed; then F has a fixed point $\bar{x} \in X$.

Theorem 2. (Park, 1997b). Let X be an admissible convex subset of a topological vector space E, and let $T \in \beta(X,X)$. If T is compact and closed, then T has a fixed point in X.

Applying Lemmas 3 and 4, we give a simple proof of Theorem 2.

Proof. By Lemma 3, $T \in KKM(X,X)$ is compact and closed. Then, by Lemma 4, T has a fixed point. □

Corollary 1. (Park, 1997a). Let X be a nonempty convex subset of a locally convex topological vector space E. Then, any closed compact map $F \in \beta(X,X)$ has a fixed point.

Proof. Since X is a convex subset of a locally convex topological vector space, it follows that X is admissible. Then, Corollary 1 follows from Theorem 2. □

Lemma 5. (Mehta *et al.*, 1994). Let D be a nonempty closed convex subset of a topological vector space E, and let $T:D \to 2^D$ be a Φ-condensing map. Then, there exists a nonempty compact convex subset K of D such that $T(K) \subseteq K$.

Theorem 3. Let X be an admissible closed convex locally convex subset of a topological vector space E. If $T \in \beta(X,X)$ is a closed Φ-condensing map, then T has a fixed point in X.

Proof. By lemma 5, there exists a nonempty compact convex subset K of X such that $T(K) \subseteq K$. It follows from Theorem 1 that $T|_K \in \beta(K,K)$. Since K is a compact convex locally convex subset of a topological vector space, K is admissible. Since T is closed, it follows from Theorem 2 that $T|_K$ has a fixed point. This completes our proof. □

Corollary 2. (Lin and Yu, 1999). Let X be a compact convex locally convex subset of a topological vector space E. Then, any closed Φ-condensing map $T \in KKM(X,X)$ has a fixed point.

Proof. It follows Lemma 3 that $T \in (X,X)$ is a closed

Φ-condensing map. Then, Corollary 2 follows immediately from Theorem 3. □

Remark 2. If X is a nonempty closed convex subset of a locally convex topological vector space, then Theorem 3 reduces to Theorem 7 (Park, 1997a).

We can use a result of Ben-El-Mechaiekh and Deguire (1992) to get some fixed point theorems.

Lemma 6. (Ben-El-Mechaiekh and Deguire, 1992). Let X be a compact subset of a topological vector space E, let Y be a subset of a topological vector space F, and let Γ be a closed subset of $X \times Y$. Then, the following statments are equivalent:

(1) $g_r f \cap \Gamma \neq \phi$ for each $f \in C(X,Y)$;

(2) $g_r A \cap \Gamma \neq \phi$ for each $A \in A_c(X,Y)$.

Theorem 4. Let X be an admissible convex subset of a topological vector space E. Let Y be a compact subset of a topological vector space F, and let $T \in \beta(X,Y)$ be closed. Then for any $G \in A_c(Y,X)$, TG has a fixed point in Y, and GT has a fixed point in X.

Proof. Since $T \in \beta(X,Y)$, then for any $f \in C(Y,X)$, we have $fT \in \beta(X,X)$. Since T is compact and closed, so is fT. Hence, applying Theorem 2, fT has a fixed point; that is, $g_r f \cap g_r T^{-1} \neq \phi$. It follows from Lemma 6 that for each $G \in A_c(Y,X)$, $g_r T^{-1} \cap g_r G \neq \phi$. Therefore, TG has a fixed point in Y, and GT has a fixed point in X. □

Corollary 3. (Lin and Yu, 1999). In Theorem 4, if $T \in \beta(X,Y)$ is replaced by $T \in KKM(X,Y)$, then we have the same conclusion.

Proof. If $T \in KKM(X,Y)$ is closed, then it follows from Lemma 3 that $T \in \beta(X,Y)$. The conclusion follows from Theorem 4. □

Theorem 5. Let X be an admissible convex subset of a topological vector space E, let Y be a compact subset of a topological vector space F, let Z be a subset of a topological vector space L and let $T \in \beta(X,Y)$ be closed. Then, for any $G \in A_c(Y,Z)$, $GT \in \beta(X,Z)$.

Proof. For each polytope P in X and $f \in C(Z,P)$, we have $fG \in A_c(Y,P)$. Since $T \in \beta(X,Y)$ is closed, it follows from Theorem 1 that $T|_p \in \beta(P,Y)$ is closed. Therefore, all the assumptions in Theorem 4 are satisfied. Then, $fGT|_p$ has a fixed point, and this completes our proof. □

Lemma 7. (Aubin and Cellina, 1994). Let X and Y be two topological spaces, and let $F{:}X \to 2^Y$ be u.s.c. with compact values from a compact space X to Y. Then, $F(X)$ is compact.

Corollary 4. (Lin and Yu, 1999). Let X be an admissible convex subset of a topological vector space E, let Y be a compact subset of a topological vector space F, let Z be a subset of a topological vector space L and let $T \in KKM(X,Y)$ be closed. Then, for any $G \in A_c(Y,Z)$, $T \in KKM(X,Z)$.

Proof. Clearly, that $T \in KKM(X,Y)$ is closed implies $T \in \beta(X,Y)$; then, by Theorem 5, $GT \in \beta(X,Z)$. Furthermore, for each $G \in A_c(Y,Z)$, G is u.s.c. with compact values. Note that $GT(X) \subseteq G(\overline{TX})$; hence, by the compactness of T and Lemma 7, we have that GT is compact. Therefore, by Lemma 3, $GT \in KKM(X,Z)$. □

Similar to the argument in Theorem 5, we have the following theorem.

Theorem 6. Let X be a convex space, let Y be an admissible convex subset of topological vector space E, let Z a topological space, and let $G \in A_c(X,Y)$. if $T \in \beta(Y,Z)$ is closed, then $TG \in \beta(X,Z)$.

Next, we can obtain a quasi-equilibrium theorem. Theorem 2 has an equivalent formulation as a quasi-equilibrium theorem as follows:

Theorem 7. Let X be an admissible convex subset of a topological vector space E. let $f{:}X \times X \to R$ be a u.s.c. function, and let $S{:}X \to 2^X$ be a compact closed map. Suppose that

(1) for each $x \in X$, the function M defined on X by

$$M(x) = \mathrm{Max}_{y \in S(x)}\, f(x,y)$$

is l.s.c., and that

(2) for each $x \in X$, $A \in \beta(X,X)$, where

$$A(x) = \{y \in S(x){:}f(x,y) = M(x)\}.$$

Then, there exists an $\bar{x} \in X$ such that

$$\bar{x} \in S(\bar{x}) \text{ and } f(\bar{x},\bar{x}) = M(\bar{x}).$$

Proof. We first want to show that $g_r A$ is closed in $X \times X$. Let $(x_\alpha,y_\alpha) \in g_r A$ and $(x_\alpha,y_\alpha) \to (x_0,y_0)$. Then, $f(x_0,y_0) \geq \overline{\lim}_\alpha f(x_\alpha,y_\alpha) = \overline{\lim}_\alpha M(x_\alpha) \geq \underline{\lim}_\alpha M(x_\alpha) \geq M(x_0)$. Since $g_r S$ is closed in $X \times X$, $y_\alpha \in S(x_\alpha)$ implies that $y_0 \in S(x_0)$. Clearly, $M(x_0) \geq f(x_0,y_0)$. Hence, $(x_0,y_0) \in g_r A$.

Moreover, by the compactness of S, A is also compact. Therefore, it follows from Theorem 2 that A has a fixed point; that is, there exists $\bar{x}\in X$ such that $\bar{x}\in S(\bar{x})$ and $f(\bar{x},\bar{x}) = M(\bar{x})$. □

Remark 3.

(1) If $f(x,y) = 0$ for all $x,y \in X$, then Theorem 7 reduced to Theorem 2.

(2) If $A(x)$ is convex for each $x \in X$ instead of $A \in \beta(X,X)$, then by the compactness and closedness of A, we have that A is u.s.c. with compact convex values. Therefore, $A \in K(X,X) \subseteq U_c^k(X,X) \in \beta(X,X)$, and this theorem is also true.

(3) If f and S are continuous, then condition (i) holds (Theorem 4, p. 51, Aubin and Cellina, 1994).

Corollary 5. (Lin and Yu, 1999). Let X be an admissible convex subset of a topological vector space E. let $f:X\times X\to R$ be a u.s.c. function, and let $S:X\to 2^X$ be a compact closed map. Suppose that

(1) for each $x \in X$, the function M defined on X by

$$M(x) = \mathrm{Max}_{y\in S(x)}\, f(x,y)$$

is l.s.c. ; and suppose that

(2) for each $x \in X$, $A \in KKM(X,X)$, where

$$A(x) = \{y \in S(x): f(x,y) = M(x)\}.$$

Then, there exists an $\bar{x}\in X$ such that

$$\bar{x} \in S(\bar{x}) \text{ and } f(\bar{x},\bar{x}) = M(\bar{x}).$$

Proof. Following the argument used in Theorem 7, A is closed. Hence, it follows from Lemma 3 that $A \in \beta(X,X)$; then, by Theorem 7, we complete our proof. □

Theorem 3 also can be reformulated in the form of a quasi-equilibrium theorem as follows:

Theorem 8. Let X be a closed convex locally convex subset of a topological vector space E. Let Φ be a measure of noncompactness on E, let $f:X\times X\to R$ be a u.s.c. function and let $S:X\to 2^X$ be a Φ-condensing closed map with compact values. Suppose that

(1) for each $x \in X$, the function M defined on X by

$$M(x)=\mathrm{Max}_{y\in S(x)}\, f(x,y)$$

is l.s.c. ; and suppose that

(2) for each $x \in X$, $A \in \beta(X,X)$, where

$$A(x) = \{y \in S(x): f(x,y) = M(x)\}.$$

Then, there exists an $\bar{x}\in X$ such that

$$\bar{x} \in S(\bar{x}) \text{ and } f(\bar{x},\bar{x}) = M(\bar{x}).$$

Proof. As in the proof Theorem 7, A is closed. Now, we want to show that A is a Φ-condensing map. Suppose that $D \subseteq X$ and $\Phi(D) \le \Phi(A(D))$. Then $\Phi(D) \le \Phi(A(D)) \le \Phi(S(D))$. Since S is Φ-condensing, we have $\Phi(D) = 0$; hence, A is Φ-condensing. Therefore, it follows from Theorem 3 that A has a fixed point. This completes our proof. □

Remark 4.

(1) If $f(x,y) = 0$ for all $x,y \in X$, then Theorem 8 reduces to Theorem 3.

(2) If f and S are continuous, then condition (i) holds (Theorem 4, p. 51, Aubin and Cellina, 1994).

(3) If $A \in KKM(X,X)$, then by the closedness of A, this theorem also holds.

From the above theorems, we can deduce two generalized quasi-equilibrium theorems as follows:

Theorem 9. Let X be an admissible convex subset of a topological vector space E. Let $T:X\to 2^X$ admit continuous selection, let $f:X\times X\times X\to R$ by a u.s.c. function, and let $S:X\to 2^X$ be a compact closed map. Suppose that

(1) for each $x \in X$ and $u \in T(x)$, the function M defined on X by

$$M(x) = \mathrm{Max}_{y\in S(x)}\, f(x,u,y)$$

is l.s.c.; and suppose that

(2) for each $x \in X$ and $u \in T(x)$, $A \in \beta(X,X)$, where

$$A(x) = \{y \in S(x): f(x,u,y) = M(x)\}.$$

Then, there exists an $\bar{x},\bar{y}\in X$ such that

$$\bar{x} \in S(\bar{x}),\ \bar{y} \in T(\bar{x}) \text{ and } f(\bar{x},\bar{y},\bar{x}) = M(\bar{x}).$$

Proof. Since T admits continuous selection, there exists a continuous function $h:X\to X$ such that $h(x) \in T(x)$ for each $x \in X$. We define $\upsilon:X\times X\to R$ by $\upsilon(x,y) = f(x,h(x),y)$ for all $x,y \in X$. Hence, by the assumption, we have that $M(x) = \mathrm{Max}_{y\in S(x)}\upsilon(x,y)$ is l.s.c. and that $A(x) = \{y \in S(x): \upsilon(x,y) = M(x)\} \in \beta(X,X)$. Therefore, by Theorem 7, there exists an $\bar{x}\in X$ such that $\bar{x} \in S(\bar{x})$ and $\upsilon(\bar{x},\bar{x}) = M(\bar{x})$. We may let $\bar{y} = h(\bar{x}) \in T(\bar{x})$, and this completes our proof. □

Theorem 10. Let X be a closed convex locally convex subset of a topological vector space E. Let Φ be a measure of noncompactness on E, let $T{:}X\to 2^X$ admit a continuous selection h, let $f{:}X\times X\times X\to R$ be a u.s.c. function and let $S{:}X\to 2^X$ be a Φ-condensing closed map with compact values. Suppose that

(1) for each $x \in X$, the function M defined on X by

$$M(x) = \mathrm{Max}_{y\in S(x)}\, f(x,h(x),y)$$

is l.s.c.; and suppose that

(2) for each $x \in X$, $A \in \beta(X,X)$, where

$$A(x) = \{y \in S(x){:}f(x,h(x),y) = M(x)\}.$$

Then, there exists an $\bar{x},\bar{y} \in X$ such that

$$\bar{x} \in S(\bar{x}),\ \bar{y} \in T(\bar{x}) \text{ and } f(\bar{x},\bar{y},\bar{x}) = M(\bar{x}).$$

Proof. Applying Theorem 8 and following the argument in Theorem 9, we complete the proof of Theorem 10. □

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較佳、可允許多值映射之研究

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摘　要

本研究主要探討在拓樸線性空間凸性可允許子集合上較佳、可允許多值映射之固定點，除此之外我們亦探討此映射之性質，我們利用此結果建立廣義、擬平衡定理。